4th International Conference on Computational and Mathematical Biomedical Engineering – CMBE2015
29 June - 1 July 2015, France
P. Nithiarasu and E.Budyn (Eds.)

A MESHLESS METHOD BASED ON THE MODIFIED MOVING LEAST SQUARES FOR SOFT TISSUE DEFORMING

Habib Chowdhury[1], Grand Joldes, Adam Wittek, Barry Doyle, Elena Pasternak, and Karol Miller
[1] School of Mechanical and Chemical Engineering, The University of Western Australia,
21355123@student.uwa.edu.au

SUMMARY

This paper assesses the interpolation capabilities of the Modified Moving Least Squares (MMLS) shape functions. The proposed meshless method based on MMLS is used for a brain deformation simulation in 2D. The results are compared with the commercial finite element software ABAQUS. The simulation results demonstrate the superior performance of the MMLS over classical MLS with linear basis functions in terms of accuracy of the solution.

Key words: *Meshless Method, Modified Moving Least Squares, Soft Tissue Deformation*

1 INTRODUCTION

Modelling the brain for neurosurgical simulation and neuroimage registration for image-guided surgery is a non-linear problem of continuum mechanics which involves large deformations and large strains with geometric and material non-linearities. In such cases, finite element method can fail due to element distortion. In this context, meshless methods [1] provide a better alternative where predefined mesh is not necessary. In meshless method, shape functions are important in order to approximate the unknown field functions to find the approximate solution to a problem using some arbitrarily distributed field nodes. Among other shape functions, Moving Least Squares (MLS) are preferred because the created approximation field created by this method is more smooth, continuous and consistent.

However, in order to maintain the non-singularity of the moment matrix, the classical MLS places strict requirements on the nodal distributions inside the support domain. Due to such limitations, the practical use of higher order polynomial basis in classical MLS was not so trivial for randomly distributed nodes, although they possess the capability for more accurate approximation of complex deformation fields. In this context, a modified moving least squares (MMLS) approximation has been recently developed by ISML [2,3]. This paper assesses the interpolation capabilities of the MMLS. The proposed meshless method based on MMLS is used for a brain deformation simulation in 2D. The results are compared with the commercial finite element software ABAQUS.

1.1 Modified Moving Least Squares (MMLS) method

The procedure for constructing classical MLS shape function starts with the approximation of a function $u(\mathrm{x})$, denoted by $u^h(\mathrm{x})$, which is defined by a combination of basis functions [4]. After minimization and solving the resulting systems of equations, the classical MLS approximation is obtained as:

$$u^h(\mathbf{x}) = \mathbf{P}^T(\mathbf{P}^T\mathbf{W}\mathbf{P})^{-1}\mathbf{P}^T\mathbf{W}\mathbf{u} = \sum_{j=1}^{n} \emptyset_j(\mathbf{x})u_j = \emptyset^T(\mathbf{x})\mathbf{u} \qquad (1)$$

As can be seen from equation (1), the classical MLS shape functions construction is heavily depended on the non-singularity of the moment matrix defined by $\mathbf{P}^T\mathbf{WP}$. The necessary conditions for the moment matrix to be non-singular depend on the types of basis functions used, and for higher order approximation, it demands more nodes to be put inside the support domain which results in more computational cost. Although the higher order polynomial basis has better approximation and convergence properties, these restrictions prevent the practical use of such functions.

In this context, Joldes et al. [2] developed a modified MLS with second order polynomial basis. Some vectors of positive weights for the additional constraints were added to error functional. In this approach, the modified approximant in obtained as:

$$\bar{u}^h(\mathbf{x}) = \mathbf{P}^T(\mathbf{P}^T\mathbf{WP}+\mathbf{H})^{-1}\mathbf{P}^T\mathbf{Wu} = \sum_{j=1}^{n}\bar{\emptyset}_j(\mathbf{x})u_j = \bar{\mathbf{\Phi}}^T(\mathbf{x})\mathbf{u} \qquad (2)$$

Where **H** is a matrix with all elements zeros except the last three diagonal entries, which are equal to $\boldsymbol{\mu}$

$$H = \begin{bmatrix} \mathbf{0}_{33} & \mathbf{0}_{33} \\ \mathbf{0}_{33} & diag(\boldsymbol{\mu}) \end{bmatrix} \qquad (3)$$

Using this technique, it can be shown that the nodal distributions which are admissible for the classical MLS with linear basis functions are also admissible for the MMLS which uses higher order quadratic basis.

2 METHODOLOGY

The approximation capability of the MMLS shape functions is assessed by comparing it with the classical MLS shape functions with linear and quadratic basis. In order to define all shape functions, a quartic spline weight function with circular domain was used. In this example, the same weights for all the additional constraints ($\mu_x{}^2=\mu_{xy}=\mu_y{}^2=\mu$) and a constant radius of influence (R) for all nodes were used. A 2D rectangular problem domain was defined and the geometry was represented using 324 nodes in both regular and irregular manner.

The following function was used for testing the approximation accuracy in 2D for different values of μ and R. The function was chosen to present a variety of behaviour in a surface.

$$u(x,y) = (1-x^2-y^2)e^{-\frac{1}{2}(x^2+y^2)} \qquad (4)$$

The approximation accuracy was determined using the root mean square error evaluated using a regular distribution of N=81*81 points:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(u(x) - u^h(x))^2}{N}} \qquad (5)$$

The results are shown in Fig.1 and Table 1.

Next, a craniotomy induced brain deformation is simulated using the Meshless Total Lagrangian Explicit Dynamics (MTLED) algorithm [5,6]. The MMLS shape functions were integrated in the algorithm. A regularized weight function was used to impose the essential boundary conditions [7]. The Young's modulus for the brain parenchyma and the tumour was set to 3000 Pa and 6000 Pa respectively and a Poisson's ratio of 0.49 was assigned for both parenchyma and tumour due to the incompressibility of the brain tissue. The interaction between skull and brain was modelled as finite sliding, frictionless contact. The skull was assumed to be rigid and the ventricles are modelled as a cavity. A variable load in terms of displacement was enforced on the nodes of the brain surface. In the meshless computation, the brain model was discretised with 707 nodes, and 4988 integration points were created from a triangular background grid with four integration points per cell. A constant influence domain (R=8) and same weights for the additional constraints (μ =10^{-7}) were used. The constitutive material laws, loading and boundary conditions were identical in both meshless and ABAQUS computations. Higher order plain strain elements with hybrid formulation were used in ABAQUS to handle the incompressibility of the soft tissues. The results are shown in Fig.2 and Table 2.

3 RESULTS AND CONCLUSIONS

3.2 MMLS Approximation capability in 2D

From the results shown in Table 1, it can be seen that as the nodal influence domain radius is decreased, the classic MLS with quadratic basis fails due to singular moment matrix. However, the MMLS with quadratic basis is stable. The approximation accuracy of MMLS is found to be better than that of classical MLS with linear basis function. Moreover, it is also noticeable that if the value of μ is decreased, the MMLS accuracy approaches the accuracy of classical MLS with quadratic basis function.

Table 1. Root mean square error (RMSE) in approximating $u(x,y) = (1 - x^2 - y^2)e^{-\frac{1}{2}(x^2+y^2)}$ using 324 nodes with varying radius of nodal influence domain, R.

Approximation method	Regular node distribution			Irregular node distribution		
	R=2.0	R=1.5	R=0.8	R=2.0	R=1.5	R=0.8
MLS, linear BF	0.0817	0.0636	0.0218	0.0861	0.0651	0.0272
MLS, quadratic BF	0.0146	0.0100	Singular M	0.0179	0.0132	Singular M
MMLS, $\mu = 0.1$	0.0220	0.0223	0.0203	0.0246	0.0125	0.0261
MMLS, $\mu = 0.01$	0.0153	0.0114	0.0139	0.0186	0.0145	0.0209
MMLS, $\mu = 0.001$	0.0147	0.0101	0.0072	0.0180	0.0133	0.0126
MMLS, $\mu = 0.0001$	0.0146	0.0100	0.0059	0.0179	0.0132	0.0099

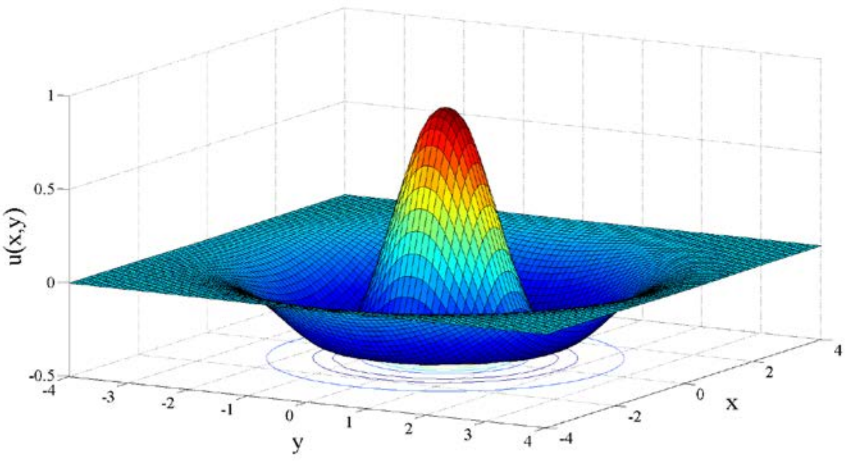

Figure 1. *Approximated function by modified MLS (μ=0.1, R=0.8) using a regular distribution of 324 nodes.*

3.2 Simulation of Brain Deformation in 2D

Table 2. Numerical details of comparison for the cases presented in Figure 5.

Case	Nodes	Elements (ABAQUS)	Integration points (Meshless)	Average difference (mm)	Maximum difference (mm)
a) Classical MLS	707	1247	4988	0.14509	0.67531
b) MMLS (μ =10^{-7})	707	1247	4988	0.12332	0.50729

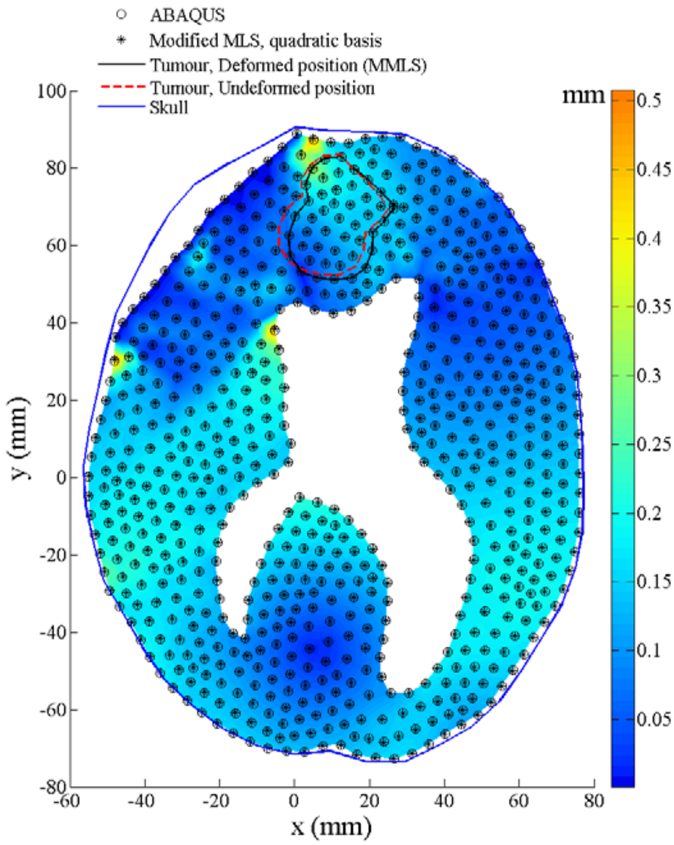

Figure 2. *Differences of the computed deformation field* in *the* brain *a) between classic MLS (linear basis) and ABAQUS; b) between modified MLS and ABAQUS.*

From the brain simulation results in Table 2, it can be seen that the maximum and average differences between MMLS and ABAQUS are found to be lower compared to those between classic MLS with linear basis and ABAQUS. Furthermore, for the given support domain radius, the classic MLS with quadratic basis failed due to the singularity of moment matrix. Therefore, it is apparent that the MMLS with quadratic basis, with the same support domain size, is stable and deliver better accuracy compared to the classical MLS with linear basis.

REFERENCES

[1] T. Belytschko, Y.Y. Lu, L. Gu. Element-Free Galerkin Methods. International Journal for Numerical Methods in Engineering, 37:229-256, 1994.

[2] G.R. Joldes, H.A. Chowdhury, A. Wittek, B. Doyle, K. Miller. Modified Moving Least Squares with Polynomial Bases for Scattered Data Approximation. UWA, Perth, WA; Report#ISML/02/2014, 17 pages, 2014.
(school.mech.uwa.edu.au/ISML/index.php/Reports)

[3] H.A. Chowdhury, G.R. Joldes, A. Wittek, B.J. Doyle, E. Pasternak and K. Miller, Implementation of a Modified Moving Least Squares Approximation for Predicting Soft Tissue Deformation Using a Meshless Method. In: Doyle, B.J., K. Miller, A. Wittek and P.M.F. Nielson (Ed's). Computational Biomechanics for Medicine: New Approaches and New Applications (6th ed), Springer NY, 2015, in press. ISBN:978-3-319-15502-9.

[4] G.R. Liu, Meshfree methods: moving beyond the finite element method, CRC press, 2010.

[5] K. Miller, G.R Joldes, D. Lance, A. Wittek, Total Lagrangian explicit dynamics finite element algorithm for computing soft tissue deformation. Communications in Numerical Methods in Engineering, 23(2):121-134, 2006.

[6] A. Horton, A. Wittek, G.R., Joldes, K. Miller, A meshless Total Lagrangian explicit dynamics algorithm for surgical simulation. International Journal for Numerical Methods in Biomedical Engineering, 26(8):977-998, 2010.

[7] T. Most, C. Bucher, A Moving Least Squares weighting function for the Element-free Galerkin Method which almost fulfills essential boundary conditions. Structural Engineering and Mechanics, 21(3):315-332, 2005.