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Numerical analysis of maximal bat performance in baseball

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Abstract

Metal baseball bats have been experimentally demonstrated to produce higher ball exit velocity (BEV) than wooden bats. In the United States, all bats are subject to BEV tests using hitting machines that rotate the bat in a horizontal plane. In this paper, a model of bat–ball impact was developed based on 3-D translational and rotational kinematics of a swing performed by high-level players. The model was designed to simulate the maximal performance of specific models of a wooden bat and a metal bat when swung by a player, and included material properties and kinematics specific to each bat. Impact dynamics were quantified using the finite element method (ANSYS/LSDYNA, version 6.1). Maximum BEV from both a metal (61.5 m/s) and a wooden (50.9 m/s) bat exceeded the 43.1 m/s threshold by which bats are certified as appropriate for commercial sale. The lower BEV from the wooden bat was attributed to a lower pre-impact bat linear velocity, and a more oblique impact that resulted in a greater proportion of BEV being lost to lateral and vertical motion. The results demonstrate the importance of factoring bat linear velocity and spatial orientation into tests of maximal bat performance, and have implications for the design of metal baseball bats.

Keywords: Baseball equipment testing; Finite element method

1. Introduction

In the United States, the introduction of durable, lightweight metal bats into youth and collegiate baseball in 1972 prompted investigation of ball exit velocity (BEV). BEV is an important quantity related both to offensive production by the batter (such as home runs) (Adair, 1997), and also to the risk of ball-impact injury to defensive players (Dick, 1999; Mueller et al., 2001). Bryant et al. (1979) first noted the performance advantage of metal bats, as measured by BEV, and the data of Greenwald et al. (2001) indicate that the gap between metal and traditional wooden bat is widening as metal bat design becomes increasingly sophisticated. Bat performance is tested experimentally using hitting machines which rotate the bat in a horizontal plane at 29.3 m/s (66 mph) against a ball projected at 31.1 m/s (70 mph). The current BEV recommendation of 43.1 ± 0.4 m/s (97 ± 1 mph) is based on the average BEV from a professional-quality wooden bat, which is the traditional hitting implement in the game and still used by all professional players (NCAA news release, 27 September 1999). Such test protocols, however, do not account for important factors in the dynamics of the impact such as bat mass and moment of inertia, and the varying pre-impact velocity and spatial orientation of different bats. As such, the results may not reflect the performance capabilities of the bat when swung with the complex combination of translation and rotation used by high-performance players (Nicholls et al., 2003).

While much theoretical interest has been devoted to the performance of baseball bats and balls and the dynamics of impact between the two (Nathan, 2000; Cross, 1999; Brody, 1990; Van Zandt, 1992; Watts and Baroni, 1989), the question of maximum BEV attainable from bats swung by players in a game environment has

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not been addressed. In this paper, explicit finite element analysis (FEA) was used to approximate the impact response of the ball and the maximum obtainable BEV. A powerful mathematical procedure for solving systems of differential equations (Bathe, 1996), FEA was utilized in order to account for the friction, irregularly shaped contact area, nonplanar motion and multi-directional forces inherent in the problem. In using FEA, the need for simplifications used in previous analysis of bat–ball impact, such as linear elastic material properties and assumptions of planar motion, were obviated. FEA has previously been used to evaluate baseball bat performance using models replicating the experimental hitting machine setup (Shenoy et al., 2001; Smith, 2001; Smith et al., 2000; Mustone and Sherwood, 1998). However, bat kinematics typical of those seen in the field have not been included in any previous analysis. The purpose of this research was therefore to develop a model of bat–ball impact based on the 3-D kinematics of bats of different design, and quantify the factors important in production of BEV.

2. Methods

2.1. Physical model: baseball bats and the baseball

3-D models of two baseball bats were developed using ANSYS/LS-DYNA FEA software (version 6.1; LSTC, Livermore, CA.). Geometry was obtained from one aluminium alloy bat (Easton BE-811) and one wooden bat (Easton Redline Pro Stix 271) of similar length and mass (Table 1). Data from calliper measurements made of the internal and external geometry at 168 intervals along the bat length were input as Cartesian (x, y, z) coordinates in LSDYNA. The shapes were generated using spline functions to avoid discontinuities resulting from stepped cross-sectional properties (see Fig. 1). The wooden bat was modelled as a homogeneous solid, discretised into 9800 eight-node SOLID-164 hexahedral elements. These elements were used to prevent meshing and hourglassing problems in the contact region, and improve the accuracy of the solution (ANSYS/LS-DYNA Theoretical Manual, 2002). The metal bat was

Table 1
Geometric and material properties of the wooden and metal baseball bats used as the basis for this mathematical model of bat–ball impact

		Wooden bat		Aluminium alloy bat	
		Experimental	LSDYNA	Experimental	LSDYNA
Mass	kg, oz	0.840 (29.6)	0.872	0.805 (28.4)	0.840 (29.6)
Length	m, in	0.835 (32.8)	0.835	0.834 (32.9)	0.834 (32.9)
Density	kg/m^3	600	600	2400	2400
Young's modulus	Pa	1.22E + 10	1.22E + 10	7.00E + 10	7.00E + 10
Poisson's ratio		0.371	0.371	0.300	0.300
	m, % length		62		57
Centre of mass (CM)		0.529 (63)	0.526	0.479 (57)	0.479
Diameter at widest point (barrel)	m	0.064	0.064	0.070	0.070
Wall thickness	mm				
Handle		n/a		34	
Throat		n/a		22	
Barrel		n/a		33	
Moments of inertia about bat knob					
I_{xx}	kg m^2		0.356		0.336
I_{yy} (swing moment)	kg m^2	0.329	0.309	0.269	0.254
I_{zz} (polar moment)	kg m^2	0.032	0.114	0.057	0.00883
I_{xy}	kg m^2		−0.005		−1.35
I_{yz}	kg m^2		0.145		0.128
I_{zx}	kg m^2		0.0093		0.00232
Moments of inertia about CM					
I_{xx}	kg m^2		0.00433		0.00683
I_{yy}	kg m^2		0.00376		0.00518
I_{zz}	kg m^2		0.00140		0.00182
I_{xy}	kg m^2		−0.00049		−0.00012
I_{yz}	kg m^2		0.00142		0.00114
I_{zx}	kg m^2		0.00091		0.00020
Principal moments of inertia					
I_{xx}	kg m^2		0.00462		0.00684
I_{yy}	kg m^2		0.00441		0.00055
I_{zz}	kg m^2		0.00046		0.00146

Geometric values were obtained experimentally. Material values for wood (northern white ash, *Fraxnius americanus*) and aluminium alloy were taken from published tables.

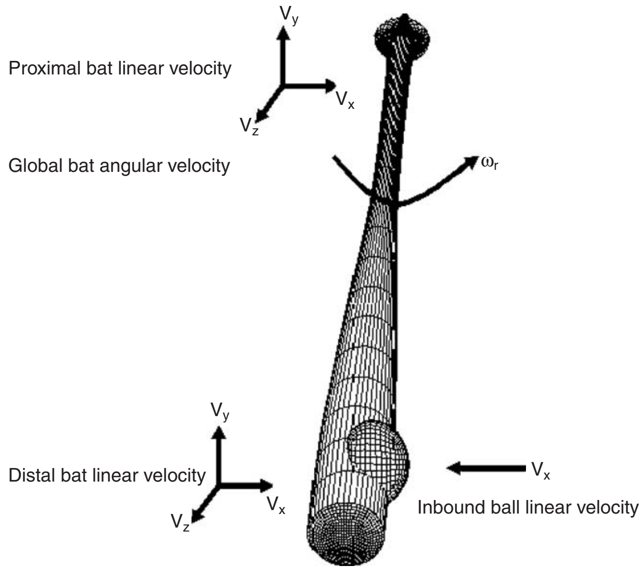

Fig. 1. Schematic diagram of baseball and baseball bat, showing finite element mesh and initial conditions imposed on the proximal and distal extremities of the bat, and the baseball.

represented as a hollow tube meshed with 12,974 hexahedral elements to accommodate the variation in wall thickness along its length (3.3–6.1 mm). Areas where there may have been irregular thickening (e.g. the knob–handle interface) were not modelled and may have been a minor source of inaccuracy. Bat orientation with respect to both vertical and horizontal planes 0.005 s prior to impact (Table 1) was obtained from digitised 3-D data collected from 17 high-performance hitters using two 200 Hz video cameras (Nicholls et al., 2003). In LSDYNA, the orientation of each bat model was achieved using sequential rotations of the global working plane from the default Cartesian X–Y orientation.

The LSDYNA *Rigid* material model was assigned to each bat. Williams (1994) indicated rigid models are appropriate when:

- bat deformation is negligible compared to its overall motion (Assumption 1);
- the collision duration is much shorter than the time taken for an impulse to propagate to the hitter's hands and back (Assumption 2).

Using rigid bodies to define stiff parts in FEA also reduces the computation time for an explicit analysis (ANSYS/LS-DYNA Theoretical Manual, 2002). The material properties were therefore of interest only to ensure the inertial properties of each bat model reflected those of real bats (material parameters for northern white ash wood and aluminium are given in Table 1). To verify the appropriateness of the assumptions and to investigate any dependence of BEV on bat materials, a check was made by simulating bat–ball impact from wooden and aluminium bats under identical initial conditions of pre-impact bat linear and angular velocity, and 3-D spatial orientation (see Section 2.2.1.).

Baseballs are made from a hard cork or rubber core wound in grey and white wools, covered in two hourglass-shaped pieces of cowhide seamed together by a single row of 216 raised stitches. Regulation baseballs have a circumference of approximately 23 cm and a mass of about 150 g. In this analysis, the baseball was approximated as a homogeneous solid sphere of radius 36 mm, meshed with 2000 SOLID164 hexahedral elements. A linear viscoelastic model was used to account for the time-dependent response of the ball to impact:

$$G(t) = G_\infty + (G_0 - G_\infty)e^{-\beta t} \tag{1}$$

Quasi-static uniaxial compression experiments to 50% of ball diameter were conducted on seven models of baseballs to obtain force–displacement data. Due to the nonlinearity of this relationship and the difficulty of quantifying large-deformation material behaviour using closed-form equations, implicit FEA was used to fit a value for G_∞ (relaxed shear modulus) to the data from the stiffest baseball. G_0 (the instantaneous modulus) was obtained through simulation of the impact of a ball on a vertical rigid wall at five velocities ranging from 13.2 to 40.2 m/s, and comparing the ratio of inbound to outbound velocity (coefficient of restitution, COR) to the experimental results of Hendee et al. (1998). The decay constant β was set to 0.0007 s (the approximate duration of bat–ball impact) (Nicholls et al., 2004).

2.2. Mathematical model

2.2.1. Initial conditions

As indicated in Assumption 2, during a high-speed impact, the baseball may be in contact with the bat for as little as 1 ms, a period far less than the typical vibrational response time of a wooden or metal bat (Adair, 1997; Noble, 1998). Therefore, BEV is highly dependent on the linear velocity of the bat impact point, which in turn is affected by the distribution of mass along the long axis of the implement (swing moment of inertia) (Fleisig et al., 2002; Greenwald et al., 2001). Bat moments of inertia were experimentally determined using the pendulum technique described in Elliott and Ackland (1982) and the parallel axis theorem (Winter, 1990). These results and the inertial properties calculated by the software are given in Table 1. The simulation was conducted over a time period commencing 0.005 s after the ball last contact with the bat. The linear velocities of the proximal and distal ends of the bats, and angular velocities about an axis 10 cm proximal to the bat knob at the instant prior to impact, were obtained from the videographic analysis described

Table 2
Initial conditions of linear and angular velocity and 3-D horizontal and vertical orientation for each bat were obtained experimentally from a group of semi-professional players hitting baseballs pitched to them by experienced pitchers

		Wooden bat	Metal bat
Experimental orientation values			
Horizontal orientation	deg	−18.5	−5.7
Vertical orientation	deg	−27.8	−30.3
LSDYNA principal orientation vectors			
X		−0.256	−0.043
Y		−0.414	−0.293
Z		0.874	0.955
TIP linear velocity			
X	m/s	35.2	37.2
Y	m/s	−1.6	−0.7
Z	m/s	3.9	4.8
HANDLE linear velocity			
X	m/s	3.9	5.4
Y	m/s	1.1	1.2
Z	m/s	−0.2	1.0
Angular velocity			
X	rad/s	40.6	6.5
Y	rad/s	40.9	40.4

Two 200 Hz video cameras were used to quantify the bat kinematics (for full details, see Nicholls et al., 2003).

above (Nicholls et al., 2003) (Table 2). The inbound ball was given an initial velocity of 40.2 m/s (90 mph), typical of pitch speeds in collegiate and professional baseball (Adair, 1997).

2.2.2. Boundary conditions

In analysis of performance of hitting equipment in sport, acceleration histories, mode shapes and BEV have all been shown to be affected by the boundary conditions imposed at the proximal (grip) end of the implement (Friswell et al., 1997; Iwata et al., 1990; Jenkins and Calder, 1990; Penrose and Hose, 1999; Weyrich et al., 1989; Wicks et al., 1999). Previous FEA of bat performance have used rigidly clamped conditions at the grip to replicate the setup of experimental hitting machines (Smith, 2001; Smith et al., 2000; Mustone and Sherwood, 1998). In our study, the mass and damping effects of the player's hands were not modelled, with both ends of the bat free to translate and rotate. This assumption was appropriate given the very short impact time (Assumption 2)—if the vibrational waves arrive back at the point of impact after the ball has departed, ball motion cannot be affected by how the handle is secured.

2.3. Explicit finite element analysis of bat–ball impact

To determine maximum BEV, the analysis was restricted to impact in which the incident trajectory of the ball was perpendicular to the point of greatest momentum transfer on the bat. While the tip of the bat has the greatest linear velocity as it is furthest from the axis of rotation, a vibrational antinode exists here and impact at this point results in significant losses to bat deformation (Noble, 1998). Maximum BEV has been shown to occur in a region in the barrel close to the nodes of the lowest frequency bending vibrations (Noble, 1998). In this analysis, five impact locations on the barrel were simulated for each bat: 570, 610, 630, 650 and 670 mm distal to the bat knob. Maximum BEV was obtained for impact 650 mm from the knob for the wooden bat, and at 670 mm for the metal bat.

The LSDYNA general surface-to-surface contact algorithm was assigned as it is suitable for arbitrarily shaped contact areas with large amounts of relative sliding (ANSYS/LS-DYNA Theoretical Manual, 1999). The nodes comprising the bat and ball were designated as *contact* and *target* components, respectively. A distance of 0.004 m initially separated the components because initial penetration is not permitted in an LSDYNA explicit analysis. The friction coefficient (μ_c) of 0.2 was determined from the relative velocity of the surfaces in contact, following the guidelines by Grigoriev and Meilikhov (1997).

Due to the transient nature of the event, large deformations and nonlinear response during bat–ball impact, explicit FEA procedures were used (Bathe, 1999). Termination time was set to 0.01 s to allow for pre-impact bat and ball motion, the bat–ball impact period (approximately 1 ms), and sufficient time to capture the post-impact oscillations of the ball. The solution time step was automatically determined by LSDYNA from the relative difference in contact surface stiffness between bat and ball. The adequacy of the bat and ball meshes is a critical factor in the accuracy of the FEA solution. If the level of discretisation (i.e. the number of degrees of freedom) is too small, the model resolution will be too low to accurately represent the dynamics of the structure (Friswell et al., 1997; Khalil and Viano, 1993). Repeating the analysis with 50% finer and 50% coarser meshes assessed the adequacy of the mesh. Results were reviewed using both POST1 (the ANSYS/LSDYNA general postprocessor) and POST26, the time-history processor.

3. Results

Results of the analysis are given in Table 3. Four nodal locations in the baseball were checked for stability of BEV measurement. At surface nodes on the trailing and lateral edges of the ball, and a node midway between the ball centre and surface, BEV values were affected by the oscillation and large deformation experienced by the ball. The node at the geometric centre of the ball provided the most consistent results. The temporal instant at which BEV should be quantified has not been defined in the literature. In this study, BEV was measured 0.005 s after separation of the ball from the bat, to prevent post-impact ball vibrations affecting the result. The metal bat produced greater BEV (61.5 m/s) than the wooden bat (50.9 m/s) (Fig. 2). The results of mesh convergence studies indicate that these values were stable across three mesh densities (Fig. 3). BEV from both bats exceeded the 43.1 m/s threshold, above which bats are recommended not to be certified for commercial sale. When using identical initial conditions, both bats produced a BEV within 1% of the other bat.

The peak resultant reaction force was 12.9% greater for metal bat impacts (Fig. 4). For both bats, impact duration (the period from the first reaction force measurement between bat and ball, until reaction force was zero) was 0.0009 s. However the pattern of ball deformation, shown in Fig. 5, was different for wooden and metal bats. Peak ball deformation, which was 22.5% of the original diameter for the metal bat, and 17.2% for the wooden bat, occurred much earlier for wooden bat impact (at 44.4% of the contact period, compared to 55.6% for the metal bat).

The influence of bat moment of inertia and pre-impact velocity and orientation was demonstrated by examination of the components of BEV and reaction force (RF) for each bat type. Metal bat RF and BEV were dominated by the horizontal (x) component, as evident in Figs. 4a and 6, respectively. However, for the wooden bat, a greater proportion of BEV and RF were in the lateral (z) and vertical (y) directions. For the wooden bat, peak reaction force also occurred later in the contact period (66.7% compared to 55.6% for the metal bat) (Fig. 6).

Table 3
Variables describing the baseball response to impact with a wooden and a metal bat, and the BEV from each impact

		Wooden bat	Metal bat
BEV(res) (0.005 s post-impact)	m/s	50.9	61.5
BEV(x)	m/s	42.0	59.1
BEV(y)	m/s	10.4	8.8
BEV(z)	m/s	26.8	14.7
Impact duration	s	0.0009	0.0009
Peak reaction force (resultant)	kN	23.49	26.98
Time of peak resultant force	s	0.0006	0.0005
% impact time	%	66.7%	55.6%
Peak reaction force (horizontal - x)	kN	22.0	26.8
Time of peak x-force	s	0.0005	0.0004
% impact time	%	55.6%	44.4%
Peak reaction force (vertical - y)	kN	4.1	3.3
Time of peak y-force	s	0.0006	0.0006
% impact time	%	66.7%	66.7%
Peak reaction force (lateral - z)	kN	7.3	3.4
Time of peak z-force	s	0.0006	0.0006
% impact time	%	66.7%	66.7%
Maximum ball compression	m	0.0124	0.0162
Maximum compression	% diam	17.2%	22.5%
Time of max compression	%	44.4%	55.6%
Impulse of impact	N.s	13.2	14.8

3.1. Validation

The advent of mathematical modelling has allowed the investigation of many phenomena which occur too infrequently for easy experimental study (Bathe, 1996). This is true of the "worst-case scenario" for maximal baseball bat performance, in which the baseball is only

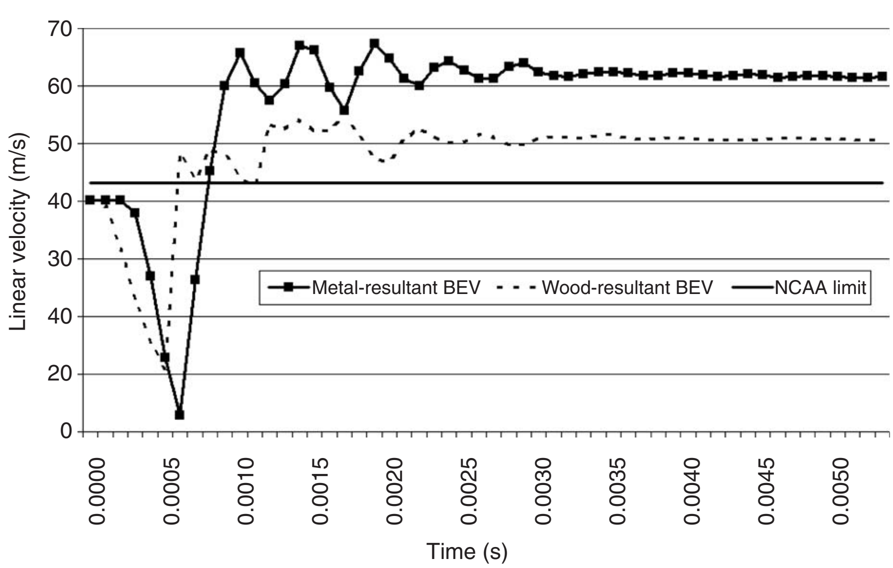

Fig. 2. BEV from wooden and aluminium alloy baseball bats for impacts at the point of zero vibration in the bat barrel.

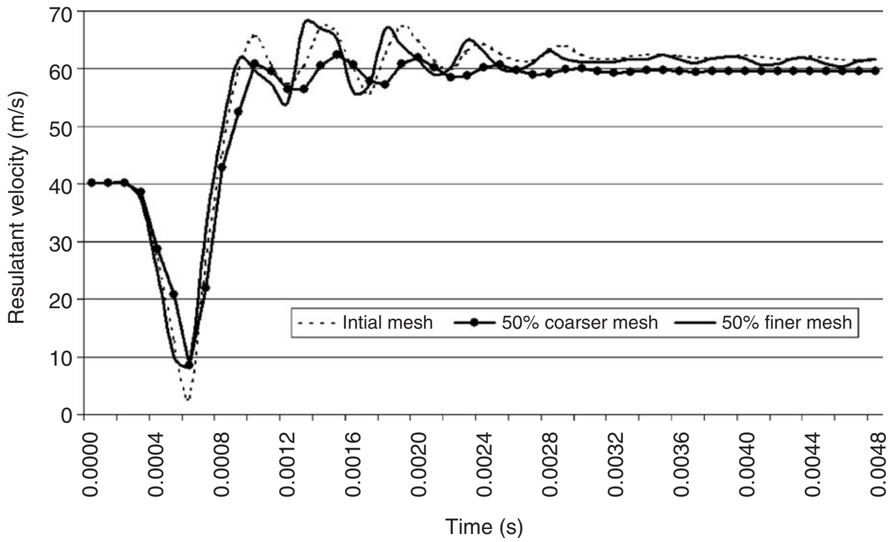

Fig. 3. Effect of mesh density on the consistency of results from this mathematical model. Convergence toward a BEV solution with less than 1% difference to the previous mesh density was considered acceptable for this analysis. Given are the results of mesh convergence testing for the metal baseball bat. The results from the wood bat analysis are the same and are not shown.

in contact with the bat for 1 ms. The 200 Hz video cameras used to experimentally quantify the velocity and 3-D orientation of each bat at the instant of impact (Nicholls et al., 2003) did not permit a detailed study of the ball and bat response during that impact time. Similarly, this ‘‘worst-case scenario’’ of ball impact at the point on the bat barrel of minimal or zero vibration cannot be easily replicated experimentally for the purposes of model validation, because it is so rare. Panjabi (1979) stated in these circumstances that validation may be achieved when ‘‘a mathematical model is combined with experimentally obtained high quality physical properties data… to form a mathematical analogue’’. Our mathematical model also satisfies the criteria of reliability and efficiency for the finite element method, as expounded by Bathe (1996). In terms of reliability, the initial and boundary conditions in our model were obtained by a series of sophisticated experiments in which the 3-D kinematics of each bat type were measured experimentally from a group of elite hitters (Nicholls et al., 2003). Likewise, the geometry of each bat was accurately and carefully measured, and the material characteristics of the baseball carefully measured and identified by us (Nicholls et al., 2004, in press). The material properties for northern white ash and aluminium assigned to each bat are considered uncontroversial and freely available from standard reference texts (Ashby, 1999; Grigoriev and Meilikhov, 1997). Results from mesh convergence studies showed the BEV results converged, thereby supporting the mesh densities employed for each analysis (Fig. 3).

4. Discussion

BEV is a measure of bat performance of interest to both hitters and defensive players. The momentum a batter can impart to the ball affects both the distance and trajectory of ball flight, and the time available to field the ball. Similarly, BEV determines the time available for an infielder to take evasive action against a ball hit directly toward him. The numerical model developed in this research accounts for the kinematics of bat motion and the time dependence of ball behaviour, and provides further insight into the performance capabilities of bats in the field. The BEV results obtained in this analysis indicate that both wooden and metal bats can produce BEV exceeding the NCAA recommendation of 43.1 m/s, with the higher pre-impact linear velocity and less oblique impact of the metal bat producing a maximum BEV of 61.5 m/s. Based on validation criteria described in Section 3, our model is both reliable and accurate, and the results are credible for analysis of this particular situation. Computer simulation was utilised to address this problem because conducting experiments equivalent to the ‘‘worst-case scenario’’ with professional players hitting a ball in the field is difficult or impossible—such experiments would essentially require the players to hit until the perfect ‘‘worst-case’’ condition occurred.

The importance of representative boundary and loading conditions for analysis of bat performance is highlighted by comparison of our BEV results to those obtained from previous explicit analysis of bat–ball impact (e.g. Shenoy et al., 2001; Smith, 2001; Smith et al., 2000;

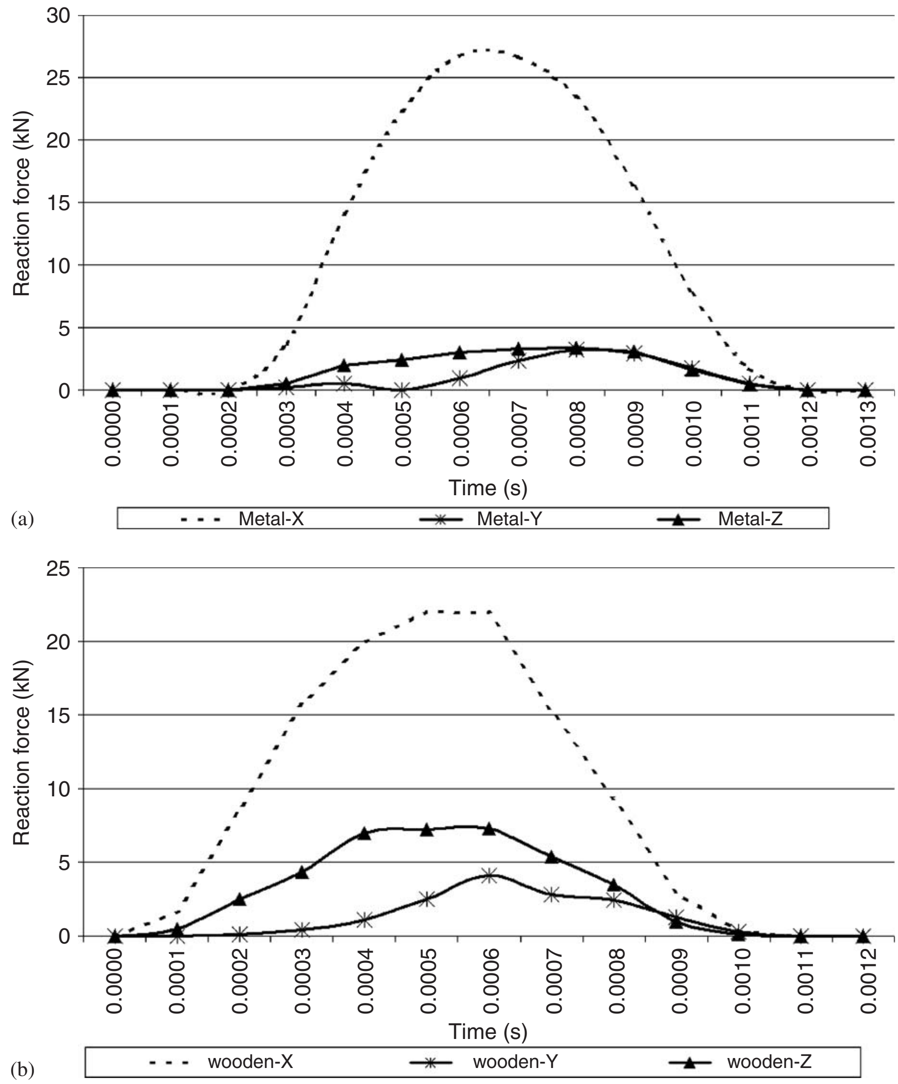

Fig. 4. (a) Pattern of reaction force during impact for a baseball impacting a metal baseball bat. (b) Pattern of reaction force during impact for a baseball impacting a wooden baseball bat.

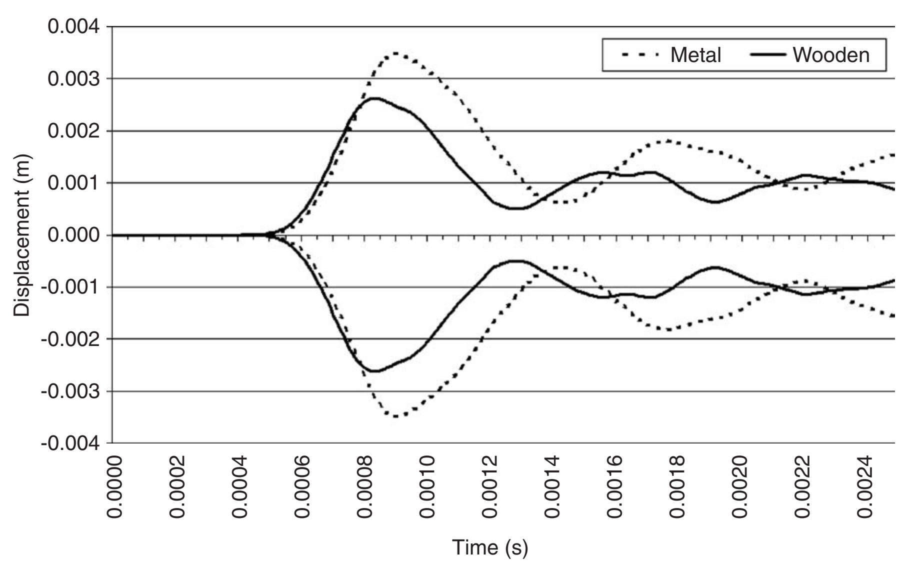

Fig. 5. Example of lateral (z) displacement of baseball surface during impact with a wooden or a metal baseball bat.

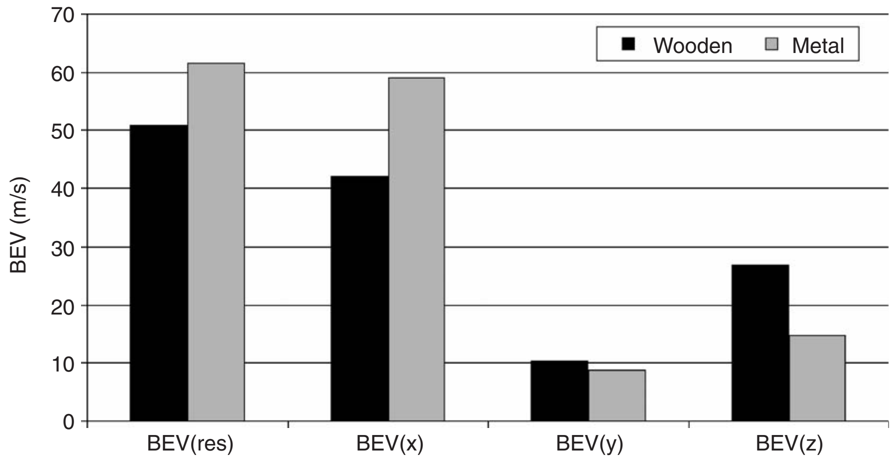

Fig. 6. Components (x, y, z) and resultant BEV 0.005 s after impact for a baseball impacting a metal and a wooden baseball bat.

Mustone and Sherwood, 1998). Lower BEV values were obtained from those bat models, which were pinned at the proximal end, with kinematics based on zero or planar rotational motion and swing velocities not representative of those seen in the field. Mustone and Sherwood (1998) obtained BEVs of 44.1 m/s for wooden bats and 48.3 m/s for metal bats when using FEA to study the performance of a hitting machine swinging bats in a horizontal plane at 31 m/s. Smith et al. (2001) obtained BEV up to 35 m/s using similar boundary conditions for wooden bats. However, bat failure at swing speeds above 22 m/s limited the loads that could be applied. While these studies were primarily intended to verify the results obtained by hitting machines and to address manufacturing issues such as bat durability, it is apparent that BEV results calculated in this manner have limited applicability to the question of maximum bat performance in the field. The assumption of constant swing velocity for bats of different weight distribution immediately introduces errors into the results. From our analysis, the complex combination of translational and rotational motion used in the swing of a high-performance player can produce much higher BEVs than those evidenced from hitting machines.

The simplifications inherent in the use of hitting machines have led to the development of an "notionally-acceptable BEV" recommendation for bats (43.1 m/s) which may not reflect the capabilities of wooden or metal bats in the field. Greenwald et al. (2001) showed that even high-school-aged hitters were able to achieve BEV in excess of 43.1 m/s using metal bats, which indicate the kinematics of the swing should be accounted for in tests of bat performance. Smith (2001) proposed a set of "field-like" conditions to improve the applicability of test results from hitting machines, including increasing inbound ball speed and bat angular velocity and factoring in bat inertia. Analysis using explicit FEA would allow easy control of all these variables, reduce the time spent in iterative redesign and bat prototype fabrication, and permit a more thorough understanding of mechanical behaviour related to production of BEV.

There are immediate practical applications for this research related to bat design and BEV. The lower moment of inertia of the metal bat resulted in higher BEV than the wooden bat with its proportionally heavier distal end. The results of this analysis substantiate the view that metal bats provide an offensive advantage for the batter (Thurston, 1999). Similarly, while ball-impact injuries to professional defensive players facing hitters using wooden bats have been documented (Bowman and Zoss, 1989; Zagelbaum et al., 1994), the higher incidence of such injuries in collegiate (Dick, 1999) and youth baseball (Adler and Monticone, 1996; Mueller et al., 2001) indicates that the use of metal bats may present a significant risk of injury to infield players. Our results lend support to the 1999 proposal by the NCAA Baseball Research Panel to regulate the swing moment of metal bats in order to achieve more "wood-like" BEV (NCAA news release, 12 June 1999).

The effect of modification in baseball behaviour on BEV is currently unknown, although Heald (1999) recommended reducing the current COR regulations and increasing ball "compressibility" as a method of controlling BEV. Using our model, the effect of changing the instantaneous shear (G_0) and relaxation (G_∞) properties of the baseball could be objectively quantified without the need for expensive experimentation. This remains the subject of our ongoing research.

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