Neuroimage Registration as Displacement - Zero Traction Problem of Solid Mechanics

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Abstract. We describe a problem of brain image registration during image-guided procedures using a framework of solid mechanics. We show that the registration of pre-operative MRIs on sparse intra-operative images of the brain deformed due to craniotomy can be reasonably described as pure displacement or displacement – zero traction problems of solid mechanics. We discuss available solution methods for such problems and suggest using explicit dynamics finite element scheme. We present a computational example using clinical data confirming the appropriateness and accuracy of proposed methods.

Keywords: image registration, solid mechanics, explicit dynamics, brain

1 Introduction

Examples of therapeutic technologies that are entering the medical practice now and will be employed in the future include gene therapy, stimulators, focused radiation, lesion generation, nanotechnology devices, drug polymers, robotic surgery and robotic prosthetics [1]. One common element of all of these novel therapeutic technologies is that they have extremely localised areas of therapeutic effect. As a result, they have to be applied precisely in relation to current (i.e. intra-operative) patient's anatomy, directly over specific location of anatomic or functional abnormality, and are therefore all candidates for coupling to image-guided intervention [1]. Nakaji and Speltzer [2] list the "accurate localisation of the target" as the first principle in modern neurosurgical approaches.

As only pre-operative anatomy of the patient is known precisely from scanned images (in case of the brain, from pre-operative 3D MRIs), it is now recognised that one of the main problems in performing reliable surgery on soft organs is *Registration.* It includes matching images of different modality, such as standard MRI and diffusion tensor magnetic resonance imaging, functional magnetic resonance imaging, multi-planar MRI or intra-operative ultrasound [3]. Registration procedures involving rigid tissues are now well established. If rigidity is assumed, it is sufficient to find several points such that their mappings between two co-ordinate systems are known. Registration of soft tissues remains a research problem.

State-of-the-art image analysis methods, such as those based on optical flow [4, 5], mutual information-based similarity [6, 7], entropy-based alignment [8], and block matching [9, 10], work perfectly well when the differences between images to be co-registered is not too large, and their use has brought significantly improved clinical outcomes in image-guided surgery [11].

In this contribution we would like to suggest a conceptually different approach to registration of high quality pre-operative brain images with lower resolution intra-operative ones, based on fundamental concepts of solid mechanics.

2 Brain MRI Registration as Pure Displacement and Displacement - Zero Traction Problems of Solid Mechanics

A particularly exciting application of non-rigid image registration is in intra-operative image-guided procedures, where high resolution pre-operative scans are warped onto intra-operative ones [11, 12]. We are in particular interested in registering high-resolution pre-operative MRI with lower quality intra-operative imaging modalities, such as multi-planar MRI and intra-operative ultrasound.

This problem, when viewed from the perspective of a mechanical or civil engineer, can be considered as follows: the brain, whose detailed pre-operative image is available, after craniotomy, due to a number of physical and physiological reasons, deforms (so-called brain shift). We are interested in the intra-operative (i.e. current) position of the brain, of which partial information is provided by low-resolution intra-operative image. In mathematical terms this problem can be described by equations of solid mechanics.

Consider motion of a deforming body in a stationary co-ordinate system, Figure 1.

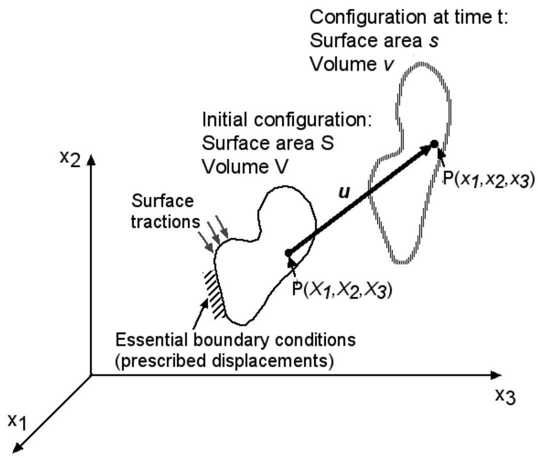

Figure 1. Motion of body in a stationary co-ordinate system. Initial configuration, described by upper case coordinates, can be considered as a high quality pre-operative image. Current, deformed configuration (described by lower-case coordinates) is unknown, however partial information is available from a lower resolution intra-operative image.

In the analysis we follow all particles in their motion, from the original to the final configuration of the body, which means that the Lagrangian (or material) formulation of the problem is adopted. Motion of the system sketched in Figure 1 can be described by differential equations of motion

$$\boldsymbol{\tau}^{ij}_{,i} + \rho F_i = 0, \qquad (1)$$

where τ denotes (Cauchy) stress, ρ is a mass density, F_i is a body force per unit mass in direction i (this includes inertial effects), and coma indicates covariant differentiation with respect to the deformed configuration. Einstein summation convention was used. Differential equations (1) must be supplemented by formulae describing the mechanical properties of materials, relating the stress to the deformation field, as well as by appropriate boundary conditions [13].

Boundary conditions may prescribe kinematic variables such as displacements and velocities (essential boundary conditions) or tractions (natural boundary conditions, these also include point forces). It should be noted that "boundary conditions" do not have to be applied at the physical boundary of the deforming object. For an object undergoing large deformations the position of the boundary is uknown and forms a part of the solution rather than of the input. Differential equations (1) form a so-called strong formulation of the problem.

An alternative, so-called weak formulation, is given by an integral equation:

$$\int_V \tau_{ij} \delta\varepsilon_{ij} dV = \int_V f_i^B \delta u_i dV + \int_S f_i^S \delta u_i dS, \qquad (2)$$

where ε is the Almansi strain, $\int_V \tau_{ij} \delta\varepsilon_{ij} dV$ is the internal virtual work, $\int_V f_i^B \delta u_i dV$ is the virtual work of external body forces (this includes inertial effects), and

$\int_S f_i^S \delta u_i dS$ is the virtual work of external surface forces. As the brain undergoes finite deformation, current volume *V* and surface *S*, over which the integration is to be conducted, are unknown: they are part of the solution rather than input data. Therefore, appropriate solution procedures, allowing finite deformation, must be used. As in the case of strong formulation (1), integral equations (2) must be supplemented by formulae describing the mechanical properties of materials, i.e. appropriate constitutive models. However, an important advantage of the weak formulation is that the essential (displacement) boundary conditions are automatically satisfied.

Depending on the amount of information about the intra-operative position of the brain, available from intra-operative imaging modalities, brain registration can be described in mathematical terms as follows:

Case I) Entire boundary of the brain can be extracted form the intra-operative image. Mathematical description:

- known: initial position of the domain (i.e. the brain), as determined from pre-operative MRI
- known: current position of the entire boundary of the domain (the brain)
- unknown: displacement field within the domain (the brain), in particular current position of the tumour and critical, from the perspective of a surgical approach, healthy tissues.

No information of surface tractions is required for the solution of this problem. Problems of this type are called in theoretical elasticity "pure displacement problems" [14].

Case II) Limited information about the boundary (e.g. only the position of the brain surface exposed during craniotomy) and perhaps about current position of clearly identifiable anatomical landmarks, e.g. as described in [15]. No external forces applied to the boundary. Mathematical description:

- known: initial position of the domain (i.e. the brain), as determined from pre-operative MRI
- known: current position of some parts of the boundary of the domain (the brain); zero pressure and traction forces everywhere else on the boundary
- unknown: displacement field within the domain (the brain), in particular current position of the tumour and critical healthy tissues.

Problems of this type are very special cases of so called "displacement – traction problems" that have not, to the best of our knowledge, been considered as a separate class and no special methods of solution for these problems exist. In Miller [16] it was suggested to call such problems "displacement – zero traction problems".

Both Case I and Case II are inherently non-linear. As the brain during surgery undergoes finite deformations (Figure 2) the boundaries where conditions are to be applied when using the strong formulation (1) are unknown. When using the weak formulation (2) the volumes and surface over which the integration is to be conducted are unknown. The Almansi strain tensor appearing in integral equations (2) is a non-linear function of displacements. Also, the stress is a non-linear function of displacements and their history.

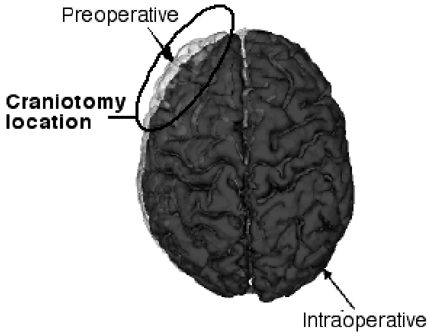

Figure 2. Comparison of the brain surface determined from images acquired preoperatively (semitransparent surface) with the one determined intra-operatively from the images acquired after craniotomy. Large deformation of brain surface due to craniotomy is clearly visible: a substantial rigid-body translation mode together with local deformations. Surfaces were determined from the images provided by Professor Simon Warfield (Brigham and Women's Hospital and Harvard Medical School, Boston, Massachusetts, USA).

3 Solution Methods

There exist a variety of methods to solve solid-mechanical problems described by differential equations (1) and integral equation (2). Finite difference method is well suited for strong form of the problem. Its main disadvantage is the difficulty with the construction of appropriate computational grids and therefore this method is rarely used in biomedical engineering applications. Boundary Element Method utilises the weak formulation (2). Unfortunately, BEM is not suited for problems involving large deformations and non-linear materials. Therefore, it is mostly applied to modelling quasi-static, small deformations [17]. Various volumetric element-free methods [18, 19] also use the weak formulation. They have been used in the image-registration context by e.g. Vigneron et al. [20] and Horton and Wittek [21]. Their significant advantage is in avoiding troublesome generation of meshes. The disadvantage is low computational efficiency that prohibits their use in intra-operative applications. By far the most commonly used is the Finite Element Method [22, 23]. All these methods can be derived from the overreaching concept of the partition of unity [24].

As the Finite Element Method is currently a dominant method used to solve solid-mechanical problems in engineering we will discuss its application to the problem described in Section 2. The fundamental difficulty in the general treatment of large deformation problems in solid mechanics is that the current configuration of a body is not known. This is an important difference compared with linear analysis, in which it is assumed that the displacements are infinitesimally small so that the configuration of the body does not change. Many researchers in the past used linear finite element method to supplement signal analysis algorithms for brain image registration and estimation of the brain shift (see e.g. [25-27]). Reference [27] uses a linear bi-phasic approach that also requires the displacements to be infinitesimal. The results obtained with linear models confirm large (up to 10 mm according to Miga et al. [28]) displacements of the brain during e.g. craniotomy-induced brain shift, therefore explicitly contradicting the assumption of infinitesimal deformations that was the basis for linear computational model development in the first place. Figure 3 shows the brain shift of Figure 2 computed with a linear finite element method.

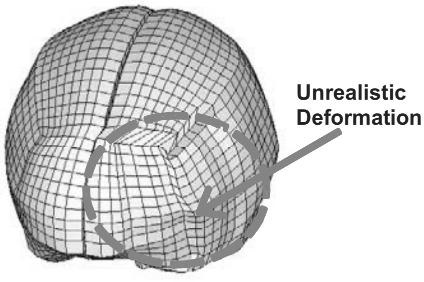

Figure 3. Unrealistic localised deformation obtained using the linear finite element analysis. The model was loaded by enforcing displacements on the brain surface in the area of craniotomy.

Various finite element procedures for finite deformation problems are described in Chapter 6 of Bathe [23]. All of them require an incremental approach, and therefore substantial pre-computations, often conducted when using linear methods, are not possible.

As for intra-operative applications the computational efficiency is essential, the most efficient solution scheme should be selected. We suggest using Explicit Dynamics Finite Element Algorithms [23, 29, 30]. The global system of equations after discretisation with finite elements, to be solved at each time step is:

$$\mathbf{M\ddot{u}_{n+1}} + \mathbf{K(u_{n+1})} = \mathbf{R_{n+1}} \tag{3}$$

where: **u** is a vector of nodal displacements, **M** is a mass matrix, **K** is a stiffness matrix non-linearly dependent on the deformation (because geometrically non-linear procedure, suitable for computing large deformations, is used), and **R** is a vector of nodal (active) forces.

Using the central difference integration scheme we obtain:

$$u_{n+1} = u_n + \Delta t_{n+1}\dot{u}_n + 1/2\Delta t_{n+1}^2\ddot{u}_n, \tag{4}$$

$$\dot{u}_{n+1} = \dot{u}_n + 1/2\Delta t_{n+1}(\ddot{u}_{n+1} + \ddot{u}_n), \tag{5}$$

$$\mathbf{K}_\mathrm{n}\mathbf{u}_\mathrm{n} = \sum_\mathrm{i} \mathbf{F_n}^{\mathbf{(i)}} = \sum_\mathrm{i} \int_{V^{(i)}} \mathbf{B}^\mathrm{T}\tau_\mathrm{n} dV \text{ , and} \tag{6}$$

$$\left(\frac{1}{\Delta t^2}\mathbf{M}\right)\mathbf{u_{n+1}} = \mathbf{R_n} - \sum_\mathbf{i} \mathbf{F_n}^{\mathbf{(i)}} - \frac{1}{\Delta t^2}\mathbf{M(u_{n-1} - 2u_n)}, \tag{7}$$

where $\mathbf{B}^\mathbf{T}$ is the strain–displacement matrix and $V^{(i)}$ is the i-th element volume.

The properties of the brain tissue are accounted for in the constitutive model by Miller and Chinzei [31] and included in the calculation of nodal reaction forces **F**. We used diagonalised mass matrix **M** that multiplies the unknown $\mathbf{u_{n+1}}$. This rendered Eq. 7 an explicit formula for the unknown $\mathbf{u_{n+1}}$. Eqs. (6) and (7) imply that computations are done at the element level eliminating the need for assembling the stiffness matrix **K** of the entire model. Thus, computational cost of each time step and internal memory requirements are small. It is worth noting that there is no need for iterations anywhere in the algorithm. This feature makes the proposed algorithm suitable for intra-operative applications.

However, the explicit methods are only conditionally stable. Normally a severe restriction on the time step size has to be included in order to receive satisfactory simulation results. For example, in car crash simulations conducted with explicit solvers the time step is usually in the order of magnitude of microseconds or even tenths of microseconds [32, 33]. The critical time step is equal to the smallest characteristic length of an element in the mesh divided by the dilatational wave speed [29, 34, 35]. Stiffness of the brain is very low [31, 36, 37]: about eight orders of magnitude lower than that of steel. Since the maximum time step allowed for stability is (roughly speaking) inversely proportional to the square root of Young's modulus divided by the mass density, it is possible to conduct simulations of brain deformation with much longer time steps than in typical dynamic simulations in engineering. The idea of explicit time integration was tested in our Laboratory by Lance [38], Wittek et al. [39] and Miller et al. [40]. The results showed that stable computations for brain meshes with ~40000 degrees of freedom are possible for time steps as large as 0.0013s.

4 Computational Example – Brain Shift Estimation

4.1 Methods

The presented example computation is a slightly modified version of the brain-shift estimation presented at MICCAI 2005 conference [41]. A three-dimensional patient specific brain mesh was constructed from the preoperative MRIs using 15036 hexahedron elements (i.e. 8-node "bricks") (Figure 4). The hexahedron finite elements are known to be the most effective ones in non-linear finite element procedures using explicit time integration.

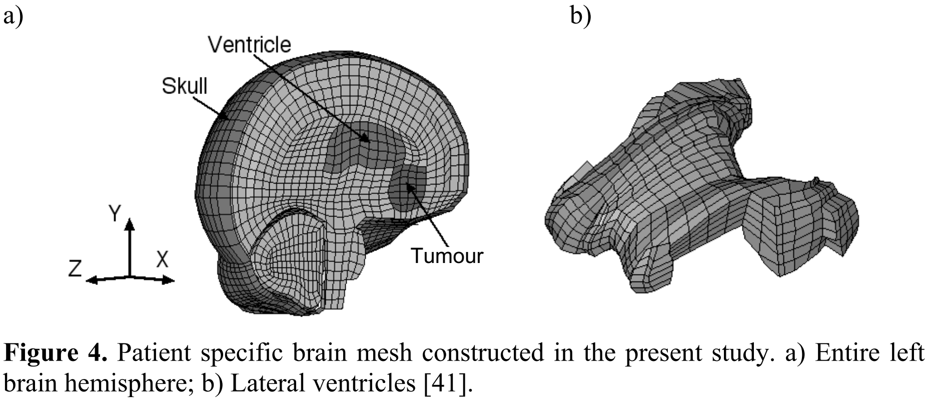

Figure 4. Patient specific brain mesh constructed in the present study. a) Entire left brain hemisphere; b) Lateral ventricles [41].

As shown by Miller and Chinzei [31, 36], the stress–strain behaviour of the brain tissue is non-linear. The stiffness in compression is significantly higher than in extension. One can also observe a strong stress – strain rate dependency. To account for these complexities, we used the model suggested in [31]:

$$W = \frac{2}{\alpha^2}\int_0^t [\mu(t-\tau)\frac{\mathrm{d}}{\mathrm{d}\tau}(\lambda_1^\alpha + \lambda_2^\alpha + \lambda_3^\alpha - 3)]\mathrm{d}\tau, \tag{8}$$

$$\mu = \mu_0[1 - \sum_{k=1}^{n} g_k (1 - e^{-\frac{t}{\tau_k}})], \tag{9}$$

where W is a potential function, λ_i's are principal stretches, μ is the instantaneous shear modulus in undeformed state, τ_k are characteristic times, g_k are relaxation coefficients, and α is a material coefficient, which can assume any real value without restrictions. The model parameters are given in Table 1.

Table 1. List of material constants for constitutive model of brain tissue, Eqs. (8) and (9), n=2. The constants were taken from ([31]

Instantaneous response	μ_0=842 [Pa]; α=-4.7
k=1	Characteristic time τ_1=0.5 [s]; g_1=0.450
k=2	Characteristic time τ_2=50 [s]; g_2=0.365

The distances between corresponding nodes of the preoperative and intra-operative cortical surfaces were calculated and used as displacement boundary conditions (i.e. prescribed nodal displacements) for the nodes located in the anterior part of the brain model surface. To define the boundary conditions for the remaining nodes on the brain model surface, the contact interface was defined between the rigid skull model and the part of the brain surface where the nodal displacements were not prescribed. No constraints were applied to the brainstem. Therefore, from the mathematical perspective our model belongs to the class of displacement – zero traction problems, and therefore no information about the nature of physical or physiological processes is required to conduct brain shift estimation.

4.2 Results

The craniotomy-induced displacements of the ventricles' and tumour's centres of gravity (COGs) predicted by the model agreed well with the actual ones determined from intra-operative MRI (Table 2). With exception of the tumour COG displacement along the Y (i.e. inferior-superior) axis, the differences between the computed and observed displacements were below 0.65 mm. Important and not unexpected feature of the results summarised in Table 2 is that the displacements of the tumour's and ventricles' COGs appreciably differed. This feature can be explained only by the fact that the brain undergoes both local deformation and global rigid body motion, which implies that non-rigid registration had to be used.

Table 2. Comparison of craniotomy-induced displacements of ventricles' and tumor's centers of gravity (COGs) predicted by the present brain model with the actual ones determined from MRIs. Directions of X, Y and Z axes are given in Figure 4a.

	Determined from MRIs	Predicted
Ventricles	Δx= 3.40 mm Δy= 0.25 mm Δz= 1.73 mm	Δx= 3.27 mm Δy= -0.43 mm Δz= 2.17 mm
Tumour	Δx= 5.36 mm Δy=-3.52 mm Δz= 2.64 mm	Δx= 4.76 mm Δy=-0.49 mm Δz= 2.74 mm

Detailed comparison of cross sections of the actual tumour and ventricle surfaces acquired intra-operatively with the ones predicted by the present brain model indicates some local miss-registration, particularly in the inferior tumour part (Figure 5). However, the overall agreement is remarkably good.

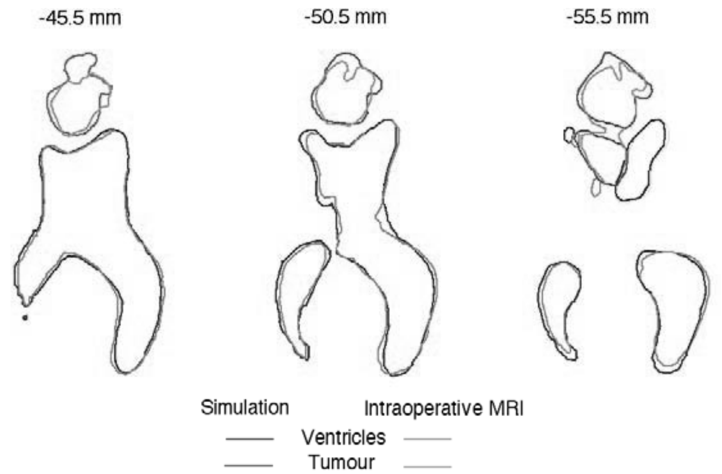

Figure 5. Comparison of contours of axial sections of ventricles and tumour obtained from the intra-operative images with the ones predicted using the presented method. Positions of section cuts are measured from the most superior point of parietal cortex (superior direction is positive).

5 Conclusions

Mathematical modelling and computer simulation have proved tremendously successful in engineering. Computational mechanics has enabled technological developments in virtually every area of our lives. One of the greatest challenges for mechanists is to extend the success of computational mechanics to fields outside traditional engineering, in particular to biology, biomedical sciences, and medicine [42]. By extending the surgeons' ability to plan and carry out surgical interventions more accurately and with less trauma, Computer-Integrated Surgery (CIS) systems could help to improve clinical outcomes and the efficiency of health care delivery. CIS systems could have a similar impact on surgery to that long since realized in Computer-Integrated Manufacturing (CIM).

In computational sciences, the most critical step in the solution of the problem is the selection of the physical and mathematical model of the phenomenon to be investigated. Model selection is most often a heuristic process, based on the analyst's judgment and experience. Often, model selection is a subjective endeavour; different modellers may choose different models to describe the same reality. Nevertheless, the selection of the model is the single most important step in obtaining valid computer simulations of an investigated reality [42].

Well-established image analysis methods applied to image registration work perfectly well when the differences between images to be co-registered are not too large. However, differences between pre- and intra-operative brain images are large enough to necessitate, in our opinion, the use of non-linear biomechanical models. It can be expected that these models supplemented by well established and appropriately chosen image analysis methods would provide a reliable method for brain image registration in the clinical setting.

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