New Finite Element Algorithm for Surgical Simulation

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Abstract

We propose an efficient numerical algorithm for computing deformations of "very" soft tissues (such as the brain, liver, kidney etc.), with applications to real time surgical simulation. The algorithm is based on the finite element method using the Total Lagrangian formulation, where stresses and strains are measured with respect to the original configuration. This choice allows for pre-computing of most spatial derivatives before the commencement of the time-stepping procedure. We used explicit time integration that eliminated the need for iterative equation solving during the time stepping procedure. The algorithm is capable of handling both geometric and material non-linearities. Stability analysis of the algorithm suggests that due to much lower stiffness of very soft tissues than that of typical engineering materials, integration time steps a few order of magnitude larger than what is typically used in engineering simulations are possible. A numerical example confirms the accuracy and efficiency of the proposed Total Lagrangian Explicit Dynamics (TLED) algorithm.

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1. Introduction

The goal of surgical simulation research is to model and simulate deformable materials for applications requiring real-time interaction. Medical applications for this include simulation-based training, skills assessment and operation planning. A surgical simulator must predict the deformation field within the organ, so that it can be displayed to the user, and the internal forces (stresses), so that reaction forces acting on surgical tools can be computed and conveyed to the user through haptic feedback.

For computational efficiency most researchers in the surgical simulation community (with two notable exceptions [1, 2]), use mathematical models based on linear elasticity. These models are incapable of providing realistic predictions of finite deformations of the tissue, because the infinitesimallity of the deformations is assumed. Linearity of the material response is also assumed. Consequently, the principle of superposition holds for linear elastic models, contradicting years of experience accumulated by researchers working on finite deformation elasticity. In the 1970s, when non-linear finite element procedures were under development, many examples of these shortcomings were published, see e.g. [3]. Therefore, in this work we present the efficient finite element algorithm, accounting for both geometric and material non-linearities, that alleviates the problems mentioned above.

2. Methods

Total Lagrangian formulation of the finite element method

Various spatial discretisation schemes are possible while using the finite element method [4]. The algorithms used by the great majority (if not all) of commercial finite element programs use the Updated Lagrangian formulation, where all variables are referred to the current (i.e. from the end of the previous time step) configuration of the system. Therefore, Cauchy stress and Almansi (or logarithmic) strain are used. The advantage of this approach is the simplicity of incremental strain description. The disadvantage is that all derivatives with respect to spatial coordinates must be recomputed in each time step, because the reference configuration is changing.

We propose to use the Total Lagrangian formulation of finite element method, where all variables are referred to the original configuration of the system. Second-Piola Kirchoff stress and Green strain are used. The disadvantage of this approach is the complicated description of finite strains resulting from the so-called initial displacement effect. However, the decisive advantage is that all derivatives with respect to spatial coordinates are calculated with respect to original configuration and therefore can be precomputed. Also, as rotation-invariant 2^{nd} Piola-Kirchoff stress is used, the necessity to rotate (Cauchy) incremental stresses before addition, present in Updated Lagrangian formulation, is eliminated. Therefore, the proposed algorithm performs significantly fewer mathematical operations in each time step.

Explicit time integration of discretised equations of motion

The present study focuses on computing the deformation field and internal forces (stresses) within a soft organ during surgical procedure. This requires application of an efficient numerical scheme when integrating equations of equilibrium (or dynamics) in time domain. Such integration can be done using either implicit or explicit methods [5, 6]. In implicit methods, the equations of dynamics are combined with the time integration operator, and the displacements are found directly. In explicit methods, on the other hand, at first the accelerations are determined from the equations of dynamics and then integrated to obtain the displacements.

The most commonly used implicit integration methods, such as the Newmarks' constant acceleration method, are unconditionally stable. This implies that their time step is limited only by the convergence/accuracy considerations. However, the implicit methods require solution of set of non-linear algebraic equations at each time step. Furthermore, iterations need to be performed for each time step of implicit integration to control the error and prevent divergence. Therefore, the number of numerical operations per each time step can be three orders of magnitude larger

than for explicit integration [5]. On the other hand, in explicit methods, such as a central difference method, treatment of non-linearities is very straightforward and no iterations are required.

However, the explicit methods are only conditionally stable. Normally a severe restriction on the time step size has to be included in order to receive satisfactory simulation results. For example, in car crash simulations conducted with explicit solvers the time step is usually in the order of magnitude of microseconds or even tenths of microseconds [7].

The critical time step is equal to the smallest characteristic length of an element in the mesh divided by the dilatational wave speed [5, 8]. Stiffness of very soft tissue is very low [9-11]: e.g. stiffness of the brain is about eight orders of magnitude lower than that of steel. Since the maximum time step allowed for stability is (roughly speaking) inversely proportional to the square root of Young's modulus divided by the mass density, it is possible to conduct simulations of brain deformation with much longer time steps than in typical dynamic simulations in engineering.

Description of finite element algorithm

Pre-computation stage:

1. Load mesh and boundary conditions
2. Precompute element Jacobians and determinants, spatial derivatives of shape functions, and linear (constant) strain-displacement matrices ($\det(\mathbf{J}), \partial\mathbf{h}, {}^{t}_{0}\mathbf{B}_{L0}$ - notation of [6] is used).
3. Compute and diagonalise (constant) mass matrix ${}^{0}\mathbf{M}$.

Initialisation:

1. Initialise nodal displacement ${}^{t}\mathbf{u} = \mathbf{0}, {}^{t+\Delta t}\mathbf{u} = \mathbf{0}$, apply load for the first time step: forces or/and prescribed displacements: ${}^{t+\Delta t}R_i^{(k)} \to R^{(k)}(\Delta t)$ or/and ${}^{t+\Delta t}u_i^{(k)} \to d(\Delta t)$

Time stepping:

Loop over elements:

1. Take element nodal displacements form the previous time step
2. Compute deformation gradient ${}^{t}_{0}\mathbf{X}$
3. Calculate full strain-displacement matrix
$$ {}^{t}_{0}\mathbf{B}^{(k)}_{L} = {}^{t}_{0}\mathbf{B}^{(k)}_{L0}\,{}^{t}_{0}\mathbf{X}^{T}. \qquad (1) $$
This matrix accounts for initial displacement effect.
4. Compute 2nd Piola-Kirchoff stress (vector) ${}^{t}_{0}\widetilde{\mathbf{S}}$ at integration points.
5. Compute element nodal reaction forces
$$ {}^{t}\hat{\mathbf{F}}^{(m)} = \int_{{}^{0}V} {}^{t}_{0}\mathbf{B}_{L}{}^{T}\,{}^{t}_{0}\widetilde{\mathbf{S}}\, d\,{}^{0}V \qquad (2) $$
using Gaussian quadrature.

Making a (time) step:

1. Obtain net nodal reaction forces at time t, ${}^{t}\mathbf{F}$.
2. Explicitly compute displacements using central difference formula
$$ {}^{t+\Delta t}u_i^{(k)} = \frac{\Delta t^2}{M_k}\left({}^{t}R_i - {}^{t}F_i^{(k)}\right) + 2\,{}^{t}u_i^{(k)} - {}^{t-\Delta t}u_i^{(k)}, \qquad (3) $$
where M_k is a diagonal entry in k-th row of the diagonalised mass matrix, R_i is an external nodal force, and Δt is the time step.

As can be seen, there is no need for solution of equations anywhere in the algorithm. Computationally most expensive parts of the algorithm are the evaluation of the full strain-displacement matrix (Point 3 of the loop over elements) and of 2nd Piola-Kirchoff stresses (Point 4 of the loop over elements). These computations must be conducted at every integration point. That is why to achieve high computational efficiency under-integrated elements should be used. For 3D

problems the most efficient elements are 8-noded hexahedra with a single integration point (for an excellent description of the implementation of this element see LS Dyna Manual [8]). For this element the integral (9) can be simply evaluated as:

$$ {}^{t}\hat{\mathbf{F}}^{(m)} = 8\, {}^{t}_{0}\mathbf{B}_{L}^{T}\, {}^{t}_{0}\tilde{\mathbf{S}}\, \det({}^{0}\mathbf{J}) \tag{4} $$

As, for stability reasons, rather small time steps are used, the strain-displacement matrix does not change much during a few time steps. Our experience shows that it is sufficient to update it every five to ten time steps. This trick provides substantial computational efficiency improvement.

Accurate evaluation of stresses at integration points is essential for accurate prediction of internal forces within the organ and computation of reaction forces acting on surgical tools. Therefore, appropriate constitutive models need to be applied. In case of hyperelastic models the 2nd Piola-Kirchoff stress can be evaluated as a derivative of the energy function with respect to Green strain. When hyper-viscoelastic models, such as the Ogden-based one developed for that brain in [11], are selected, an efficient recursive scheme should be implemented. The implementation method is explained in detail in [12].

3. Results

We simulated an indentation of an ellipsoid, Figure 1a. The elipsoid is deformed by constraining the lower half, whilst 4 nodes are displaced in the Oz direction. This is a rough approximation of indentation of the brain [13]. The boundary conditions are: Constrained nodes: $\Delta x = \Delta y = \Delta z = 0$; Displaced nodes: $\Delta x = \Delta y = 0$, $\Delta z = d(t)$. The displacements are presented for a line of nodes found at the intersection of the plane y = 0 with the surface of the ellipsoid. The reaction forces are presented for one of the displaced nodes.

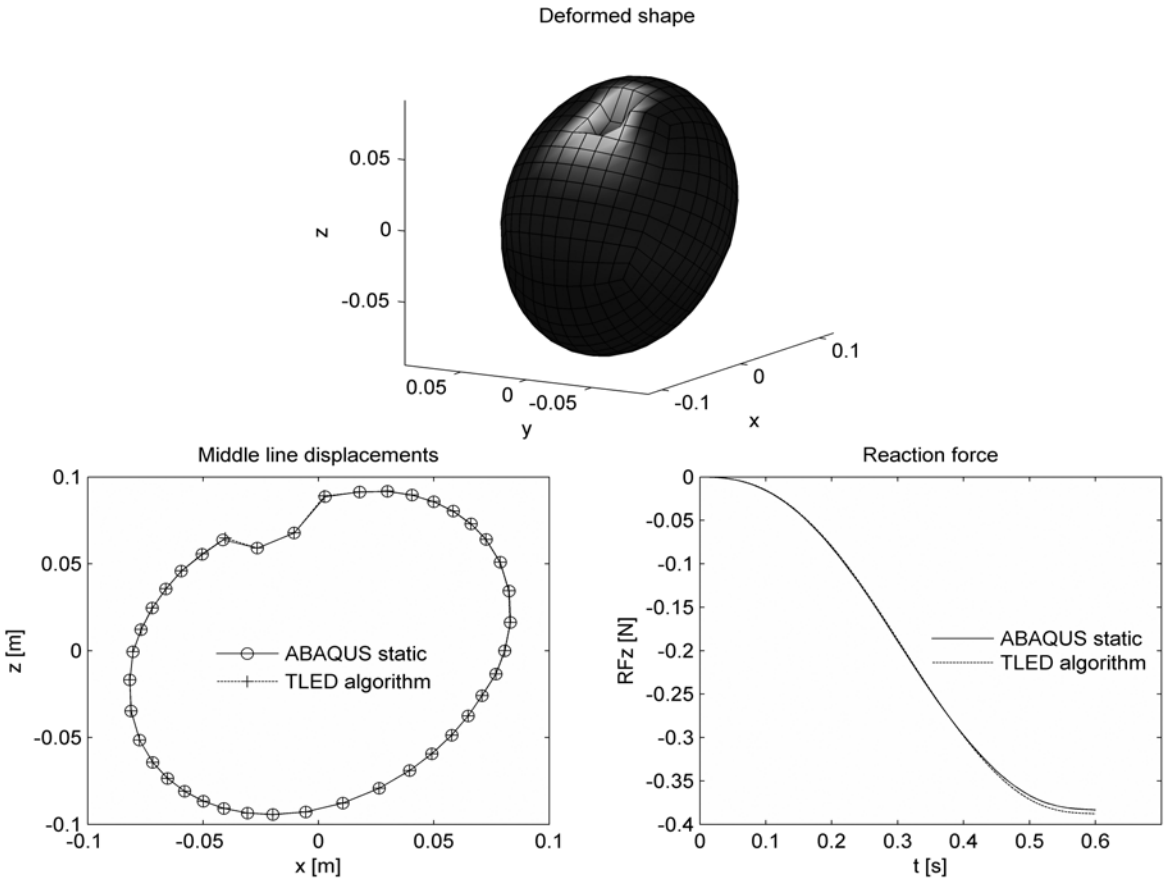

Figure 1: Indentation of an ellipsoid

The computed deformed shape of the object was compared with the one obtained using ABAQUS [14]. A Neo-Hookean material model with Young's modulus in undeformed state equal to 3000 Pa, Poisson's ratio 0.49 and mass density of 1000 kg/m^3 was used in all cases. A fully integrated

8-node linear brick, hybrid, constant pressure element was used in ABAQUS. The maximum displacement was 0.02 m in all cases and was applied using a loading curve given by:

$$d(t) = (10 - 15t + 6t^2)t^3, \quad (5)$$

where t is the relative time (varying from 0 to 1).
As can be seen in Figures 1b,c the computed results with the new algorithm agrees perfectly with the ones obtained with a reliable non-linear finite element package.

4. Discussion and conclusions

We presented an efficient algorithm based on the Total Lagrangian formulation of the finite element method combined with explicit time integration. The TLED algorithm is capable of handling geometric and material non-linearities.

The algorithm's accuracy for large strain non-linear elastic behaviour was validated using a reputable commercial software ABAQUS [14]. Our implementation of the algorithm allows the computation of a step for a 6000-element 6741-node hexahedral mesh in about 16 ms on a standard 3.2 GHz Pentium 4 system using Windows XP operating system. As our results indicate that time steps as large as of 0.001 s are indeed possible, the algorithm constitutes a step towards a real time surgical simulation.

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References:

1. Zhuang, Y., *Real-time Simulation of Physically Realistic Global Deformations*, PhD Thesis, Department of Computer Science. 2000, University of California at Berkley.
2. Picinbono, G., H. Delingette, and N. Ayache, *Non-linear anisotropic elasticity for real-time surgery simulation.* Graphical Models, 2003. 65: p. 305-321.
3. Oden, J.T. and G.F. Carey, *Finite Elements: Special Problems in Solid Mechanics*. 1983: Prentice-Hall.
4. Belytschko, T., *An Overview of Semidiscretization and Time Integration Procedures*, in *Computational Methods for Transient Analysis*, T. Belytschko and T.J.R. Hughes, Editors. 1983, North-Holland: Amsterdam. p. 1-66.
5. Belytschko, T., *A survey of numerical methods and computer programs for dynamic structural analysis.* Nuclear Engineering and Design, 1976. 37: p. 23-34.
6. Bathe, K.-J., *Finite Element Procedures*. 1996: Prentice-Hall.
7. Kirkpatrick, S.W., M. Schroeder, and J.W. Simons, *Evaluation of passenger rail vehicle crashworthiness.* International Journal of Crashworthiness, 2001. 6(1): p. 95-106.
8. Hallquist, J.O., *LS-DYNA Theoretical Manual*. 1998: Livermore Software Technology Corporation.
9. Miller, K. and K. Chinzei, *Constitutive modelling of brain tissue; Experiment and Theory.* Journal of Biomechanics, 1997. 30(11/12): p. 1115-1121.
10. Miller, K., *Biomechanics of the Brain for Computer Integrated Surgery*. 2002, Warsaw: Publishing House of Warsaw University of Technology.
11. Miller, K. and K. Chinzei, *Mechanical properties of brain tissue in tension.* Journal of Biomechanics, 2002. 35: p. 483-490.
12. Feng, W.W., *A recurrence formula for viscoelastic constitutive equations.* International Journal of Non-Linear Mechanics, 1992. 27(4): p. 675-678.
13. Miller, K., et al., *Mechanical properties of brain tissue in-vivo: experiment and computer simulation.* Journal of Biomechanics, 2000. 33: p. 1369-1376.
14. ABAQUS, *ABAQUS Theory Manual Version 5.2*. 1998: Hibbit, Karlsson & Sorensen, Inc.