Improved Linear Tetrahedral Element for Surgical Simulation

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Abstract. Automatic hexahedral mesh generation for complex geometries is still a challenging problem, and therefore tetrahedral or mixed meshes are preferred. Unfortunately the standard formulation of the linear tetrahedral element exhibits volumetric locking in case of almost incompressible materials. In this paper we propose a new linear tetrahedral element based on the average nodal Jacobian. This element improves the volumetric locking behavior of the standard formulation and requires only a small additional computation effort, providing an element that has an increased accuracy while maintaining the computational efficiency. We show how easy an existing TLED (Total Lagrangian Explicit Dynamics) algorithm can be modified in order to support the new element formulation. The performance evaluation of the new element shows a clear improvement in reaction force and displacement predictions compared to the standard formulation.

Keywords: non-locking tetrahedron**,** surgical simulation, soft tissues, Total Lagrangian formulation

1 Introduction

Finite element models needed in surgical simulation must be both fast and accurate. In order to be fast, they must use low order elements that are not computationally intensive, such as the linear tetrahedron or the linear under-integrated hexahedron. In order to be accurate, the generated mesh should approximate well the real geometry, so that the boundary conditions can be imposed accurately. Many algorithms are now available for fast and accurate automatic mesh generation using tetrahedral elements, but not for automatic hexahedral mesh generation [1-3]. Therefore, in order to automate the simulation process, tetrahedral or mixed meshes are more convenient. Unfortunately, the standard formulation of the tetrahedral element exhibits volumetric locking, especially in case of soft tissues such as the brain, that are modeled as almost incompressible materials [4].

There are a number of improved linear tetrahedral elements already proposed by different authors [5-7]. These formulations are either much more computationally intensive than the standard formulation or the volumetric lock control mechanism depends on material properties (e.g. bulk modulus), making harder the interfacing of different materials. Our volumetric lock control mechanism is computationally inexpensive and depends solely on kinematic variables, therefore making the interfacing of different materials easy.

We will show how this element formulation can be easily programmed in an existing finite element code [8] and present several simple deformation examples in order to show the increased performance of this element over the standard formulation and its computational efficiency.

2 Average Nodal Jacobian Formulation

As defined in [6], the nodal Jacobian is the ratio between the current and initial nodal volumes. The nodal volume is computed as a sum of fractions of the surrounding element volumes.

Using the nodal Jacobians, an average Jacobian can be computed for each element. Because the element Jacobian is equal to the determinant of the element deformation gradient **X**, we define a modified element deformation gradient that has the same isochoric part as the normal deformation gradient, but the volumetric part is modified so that its determinant (and therefore the volumetric deformation) is equal to the average element Jacobian.

The computation of the nodal forces (or stiffness matrix) can now be done in the usual manner, but using the modified deformation gradient instead of the normal deformation gradient for defining the strains. This way there is no need for computing the isochoric and deviatoric components of the internal forces separately, and the existing material law can be used. This will be demonstrated in the next section.

Because of the relation existing between pressure and Jacobian:

$$p = \kappa \cdot \left(\frac{V}{V^0} - 1 \right) = \kappa \cdot \left(J - 1 \right) \qquad (1)$$

this formulation is equivalent to the average nodal pressure formulation in case of a node surrounded by elements having the same bulk modulus, and therefore has the same properties (see [9] for a comparison to other formulations). Anyhow, our formulation does not depend on any material properties and the computation of the nodal forces is made using the modified deformation gradient instead of using the pressure.

3 Implementation of the Element in the Total Lagrangian Explicit Dynamic (TLED) Algorithm

The TLED algorithm is a very efficient explicit algorithm that can be used for surgical simulation, based on the Total Lagrangian formulation. The basic algorithm is presented in [8]. The modified algorithm that can use the proposed tetrahedral element presented in this paper is presented bellow. The additional steps required are marked with a (+).

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Pre-computation stage:
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1. Load mesh and boundary conditions
2. For each element compute the determinant of the Jacobian det(**J**) and the spatial derivatives of the shape functions $\partial\mathbf{h}$ (notation from [10] is used, where the left superscript represents the current time and the left subscript represents the time of the reference configuration - *0* when Total Lagrangian formulation is used).
3. Compute the diagonal (constant) mass matrix ${}^{0}\mathbf{M}$.
4. (+) Compute the initial volumes associated with each node.

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Initialization:
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1. Initialize nodal displacement ${}^{0}\mathbf{u} = \mathbf{0}, {}^{-\Delta t}\mathbf{u} = \mathbf{0}$, apply load for the first time step: forces or/and prescribed displacements: ${}^{\Delta t}R_i^{(k)} \leftarrow R^{(k)}(\Delta t)$ or/and ${}^{\Delta t}u_i^{(k)} \leftarrow d(\Delta t)$

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Time stepping:
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Loop over elements:

1. Take element nodal displacements from the previous time step
2. Compute deformation gradient ${}_0^t\mathbf{X}$ and its determinant
3. (+) Compute current element volume.

$$V_e = \det({}_0^t\mathbf{X})\, V_e^0 \quad (2)$$

(+) Compute current nodal volume.
(+) Compute the nodal Jacobian.
Loop over elements:

1. (+) Compute the average element Jacobian.
2. (+) Compute the modified deformation gradient.
3. Compute the 2nd Piola-Kirchhoff stress (vector) ${}_0^t\overline{\mathbf{S}}$ using the given material law (based on the modified deformation gradient).
4. Compute the element nodal reaction forces using Gaussian quadrature

Making a (time) step:

1. Obtain net nodal reaction forces at time t, ${}^{t}\mathbf{F}$.
2. Explicitly compute displacements using central difference formula

$$^{t+\Delta t}u_i^{(k)} = \frac{\Delta t^2}{M_k}{}^{t}({}^{t}R_i - {}^{t}F_i^{(k)}) + 2\,{}^{t}u_i^{(k)} - {}^{t-\Delta t}u_i^{(k)} \quad (3)$$

where M_k is a diagonal entry in k-th row of the diagonalized mass matrix, R_i is an external nodal force, and Δt is the time step.

3. Apply load for next step: ${}^{t+\Delta t}R_i^{(k)} \leftarrow R^{(k)}(t+\Delta t)$ or/and ${}^{t+\Delta t}u_i^{(k)} \leftarrow d(t+\Delta t)$.

The needed modifications are easy to implement and do not require major changes in the existing algorithm. The performances of the modified algorithm will be presented in the next section.

4 Simulation Results

4.1 Elements Considered

In order to asses the behavior of the presented element formulation we did a series of simple experiments using both locking and non-locking elements. The results obtained with the commercial finite element software ABAQUS [11] were used as a benchmark for our results. We used the implicit solver from ABAQUS with the default configuration. The behavior of the following elements was compared:

1. Fully integrated linear hexahedra, hybrid displacement-pressure formulation, as implemented in ABAQUS (Hexa-ABAQUS);
2. Under-integrated linear hexahedra with hourglass control, as implemented in TLED (Hexa-TLED);
3. Linear non-locking tetrahedron, implemented in TLED as presented in this paper (Tetra improved);
4. Linear standard tetrahedron from ABAQUS (Tetra standard).

4.2 Mesh

Two meshes were created for a cylinder having the diameter of 0.1 m and the height 0.1 m. The first mesh was constructed using tetrahedral elements and the second using hexahedral elements, both having similar element size (defined as the size of the element edge), using Altair HyperMesh [12]. The tetrahedral mesh had 3314 nodes and 15337 elements, whereas the hexahedral mesh had 6741 nodes and 6000 elements.

4.3 Material Model

A neo-Hookean material model was used, having the material properties similar to the brain (mass density of 1000 kg/m^3, Young's modulus in un-deformed state equal to 3000 Pa and Poisson's ratio 0.49).

4.4 Loads

In all experiments one face of the cylinder was constrained ($\Delta x = \Delta y = \Delta z = 0$) and the other face was displaced.

The following deformations were considered:

- Extension ($\Delta x = \Delta y = 0$, $\Delta z = d(t)$ on the displaced face);

- Compression ($\Delta x = \Delta y = 0$, $\Delta z = d(t)$ on the displaced face);
- Shear ($\Delta y = \Delta z = 0$, $\Delta x = d(t)$ on the displaced face);

The maximum displacement was 0.02 m in all cases and was applied using a loading curve given by:

$$d(t) = d_{\max} \cdot (10 - 15t + 6t^2) \cdot t^3 \quad (4)$$

where t is the relative time (varying from 0 to 1).

Both displacements and total reaction forces were compared for the different elements considered. The displacements are presented for a line of nodes found at the intersection of the plane y = 0 with the lateral surface of the cylinder. The total reaction force is computed as the sum of the reaction force for all nodes found on the displaced face.

4.5 Evidence of locking

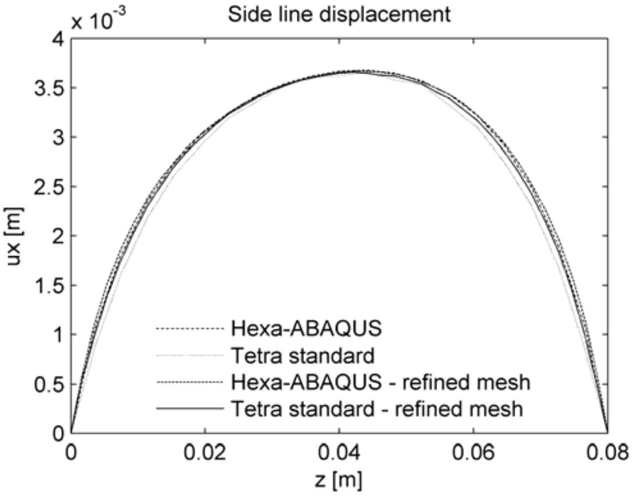
For evidencing the volumetric locking phenomena in case of almost incompressible materials we did the following experiment.

First, a material having Poisson's ratio 0.3 was considered and the simulation of the different deformations presented above were carried out. The same simulations were done for a material having Poisson's ratio 0.49 and the results compared.

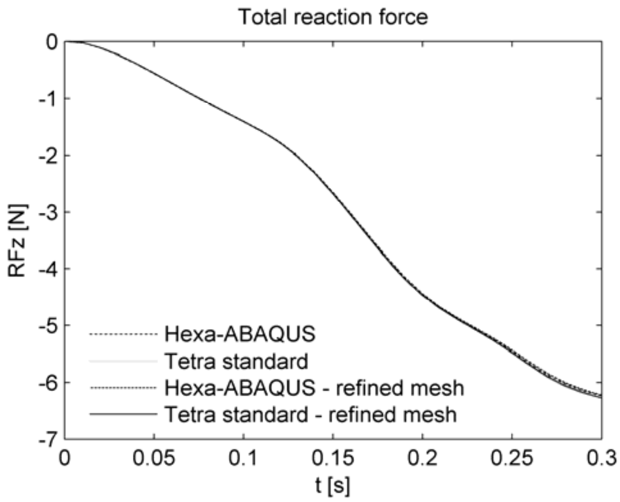
In case of low Poisson's ratio the resulting deformations and forces were very similar for any of the element types 1, 2 or 4 (see, for example, fig. 1). In case of high Poisson's ratio, the deformations and forces were very similar for the first two element types (non-locking elements) but different for the fourth element type (locking element) (see figs. 2-4).

4.6 Mesh convergence

In order to check that the difference in results obtained for different elements is not caused by other factors than volumetric locking, a mesh convergence test was done. The meshes were refined obtaining a tetrahedral mesh having 10855 nodes and 49532 elements and a hexahedral mesh having 50881 nodes and 48000 elements. The compression simulation results using the initial and the refined meshes were then compared, using the low Poisson's ratio and element types 1 and 4. As these results were very similar (less than 0.23 mm difference in displacements and less than 0.055 N difference in forces), we concluded that the meshes are convergent (see fig. 1).



a)

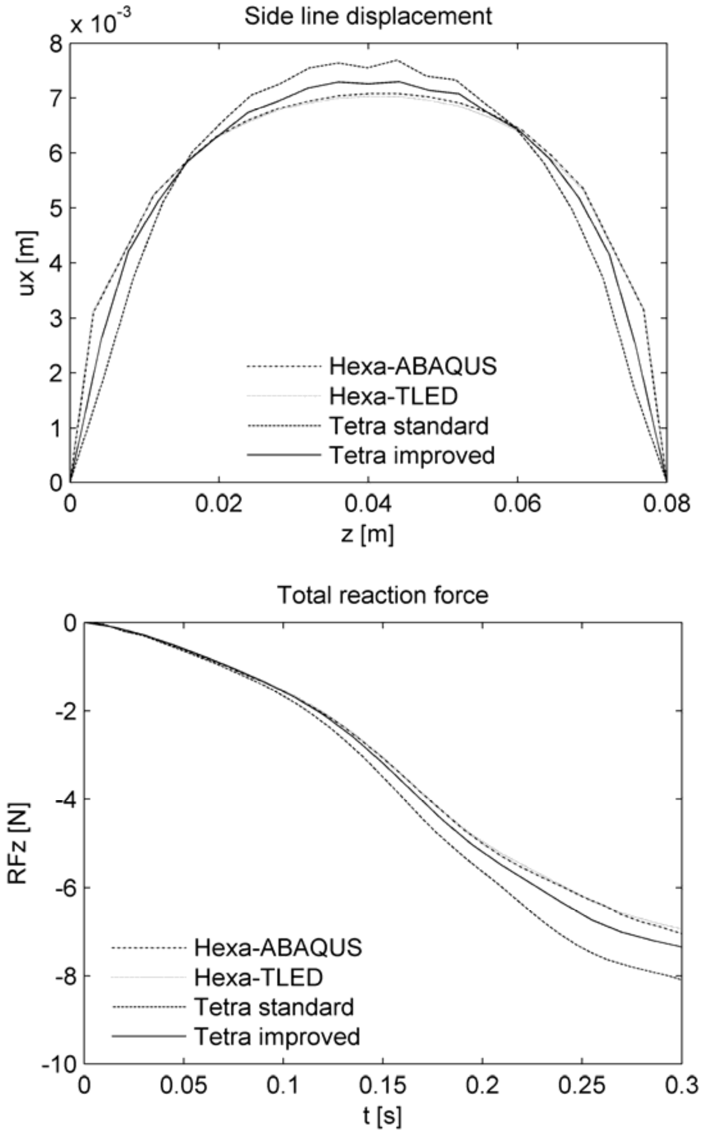


b)

Fig. 1. Displacements (a) and total reaction force (b) obtained in compression using standard hexahedral and tetrahedral elements with different mesh densities (Poisson's ratio - 0.3)

4.7 Performance of the proposed element

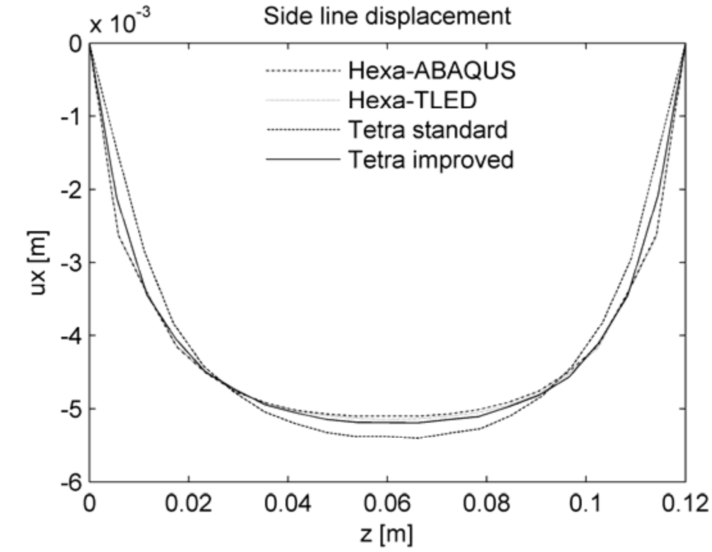
In all the studied deformation cases, the proposed element formulation provided better results than the standard formulation, both in term of displacements and forces. Some comparative results are presented bellow.



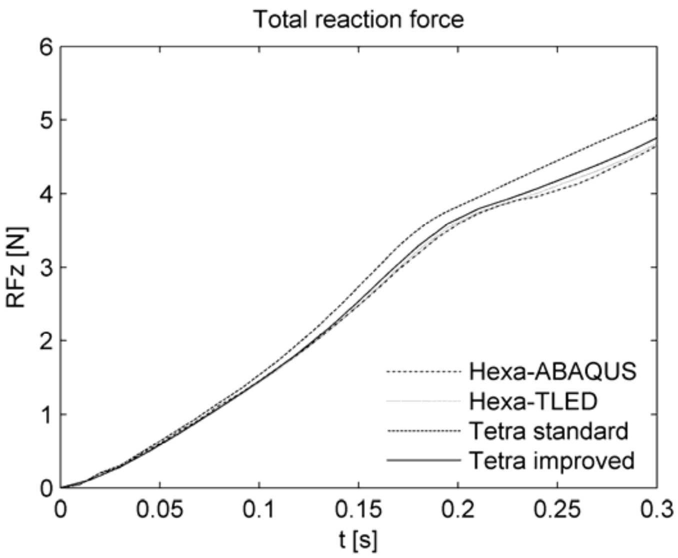
a)

b)

Fig. 2. Displacements (a) and total reaction force (b) in compression for different element types (Poisson's ratio - 0.49)



a)



b)

Fig. 3. Displacements (a) and total reaction force (b) in extension for different element types (Poisson's ratio - 0.49)

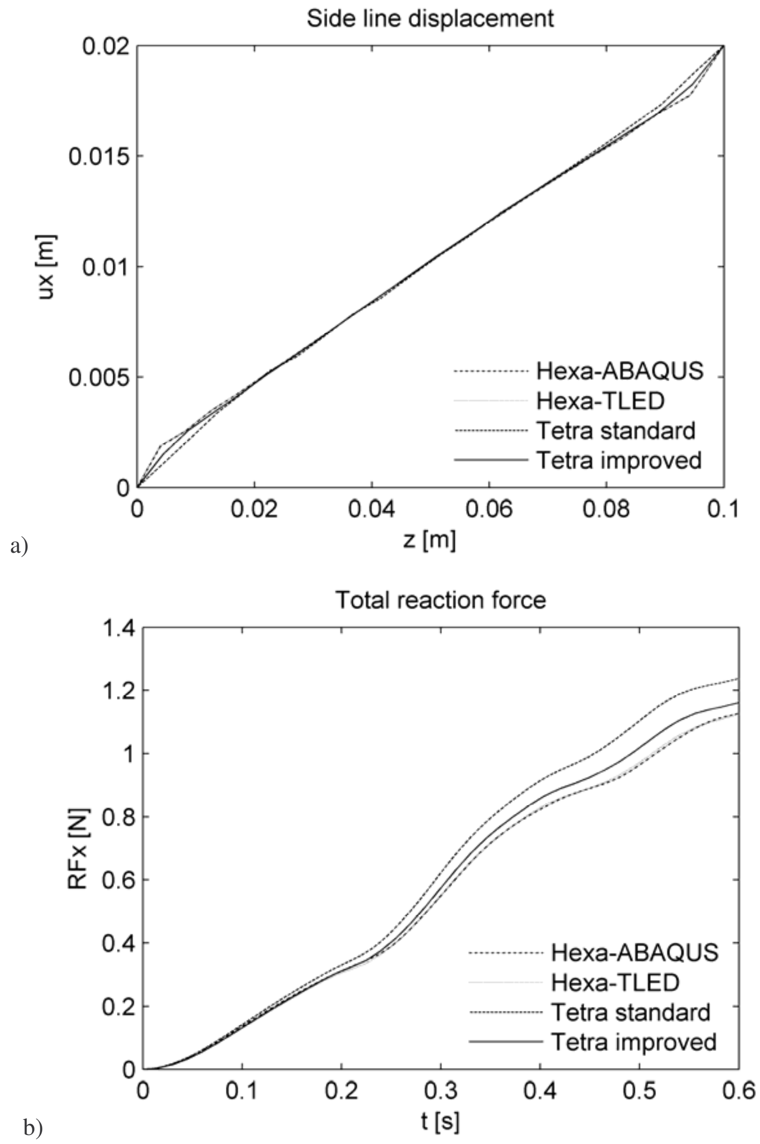


Fig. 4. Displacements (a) and total reaction force (b) in shear for different element types (Poisson's ratio - 0.49)

A quantitative comparison between the performance of the standard and the improved tetrahedral element for the three considered deformations in terms of computed reaction forces and displacements are presented in tables 1 and 2. The presented differences are computed using as benchmark data the results of the simulation made with ABAQUS using fully integrated hybrid displacement-pressure formulation hexahedral elements (type 1). A substantial improvement in the results can be observed when the improved tetrahedral element is used (the average reaction force deviations are reduced by up to 72% and the average displacement deviations are reduced by up to 70%). This can also be observed in the above figures.

Table 1. Difference in total reaction forces [N] obtained using standard and improved tetrahedral elements in case of different deformations

	Reaction force difference [N]					
Element type	Compression		Extension		Shear	
	Max.	Average	Max.	Average	Max.	Average
Tetra standard	1.192	0.482	0.445	0.204	0.143	0.062
Tetra improved	0.441	0.150	0.152	0.057	0.056	0.022

Table 2. Difference in side line displacements [mm] obtained using standard and improved tetrahedral elements in case of different deformations

	Displacements difference [mm]					
Element type	Compression		Extension		Shear	
	Max.	Average	Max.	Average	Max.	Average
Tetra standard	1.913	0.573	1.074	0.280	0.890	0.196
Tetra improved	1.272	0.236	0.458	0.083	0.515	0.081

4.8 Run-time behavior

All the simulations were done on a regular 3.2 GHz Pentium 4 desktop computer using Windows as an operating system. The code was implemented in C++.

The simulation time necessary for running 1000 steps using standard and improved tetrahedral elements with TLED, for different mesh sizes, are presented in table 3. The modified code using the improved element is about 40% more computationally expensive.

Table 3. Simulation time [s] for running 1000 steps for different mesh densities using TLED and Abaqus explicit

		Simulation time [s]		
Mesh size		TLED		Abaqus explicit
Nodes	Elements	Tetra standard	Tetra improved	Tetra standard
3314	15337	4.6	6.4	174
10855	49532	15.8	22.1	597

5. Discussions and Conclusions

A new tetrahedral element formulation based on the average nodal Jacobian is presented in this paper. This formulation uses only kinematic variables for controlling the volumetric locking, and therefore the usage of different materials and the implementation in an existing finite element code can be made without difficulties.

The performance of the proposed formulation is evaluated using the TLED algorithm against the standard tetrahedral element formulation. The differences in computed displacements and reaction forces due to volumetric locking are reduced by more than 50%.

The modified TLED code using the improved element is about 40% more computationally expensive than the standard one. This percentage might seem high, but it is only due to the fact that our definition of the standard tetrahedral element using a Total Lagrangian formulation is already very efficient. Compared with Abaqus explicit, the modified TLED implementation is still more than 20 times more efficient.

The very good performance of the under-integrated hexahedral element can also be seen from figs. 2-4. Having only one integration point, this element is very inexpensive from the computational point of view (see [8]), being a perfect candidate for real time surgical simulations. Using this type of element and the improved tetrahedral element presented in this paper for simulations using mixed meshes is a step towards complete automated patient specific surgical simulation.

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