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Modeling of Flow System Dynamics

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This paper presents non-linear and linear models of flow system of laboratory stand (semi-industrial scale) with pressurized fluidized bed boiler. The non-linear model belongs to the class of lumped parameter models. The linear model was obtained experimentally using a novel method for design of control systems in industrial plants - MIKOZ. This study has connection with comparison of various methods of mathematical modeling of flow systems frequently encountered in power equipment.

Keywords: flow system, modeling, lumped parameter model, frequency domain identification.

Introduction

The paper has connection with comparison of various methods of mathematical modeling of flow systems frequently encountered in power equipment. The concept of two totally different mathematical modeling techniques for dynamics of objects belonging to the category of "complicated systems of machines and equipment" was presented. Of primary interest here is transient process resulting in changes of global thermal and flow parameters. As an example the flow system of the laboratory stand with pressurized, fluidized bed boiler located at the Institute of Heat Engineering (IHE) of The Warsaw University of Technology was chosen.

Non-linear and linear models of this stand are presented. The non-linear model belongs to the class of lumped parameter models. The linear model was obtained experimentally using a novel method for design of control systems in industrial plants – MIKOZ[1].

The pressurized, fluidized bed laboratory stand constructed in IHE – max. power 3 MWth (semi-industrial scale) – not only allows fundamental research on combustion efficiency, desulfurisation, dedusting and process dynamics[2] but also provides an unique opportunity of verification of various mathematical modeling methods.

This possibility is connected with relatively large time constants of the installation which is equipped with original control and measurement system, intended for the real time identification and process control.

The layout of installation under investigation is shown in Fig.1. For the case analyzed we researched the flow system (air inlet-outlet) without firing the combustor.

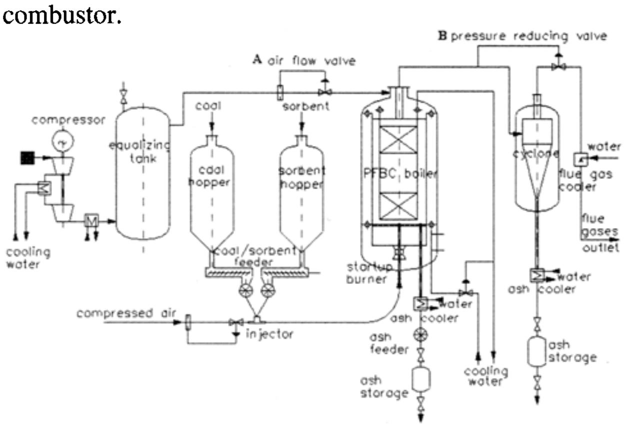

Fig.1 Layout of the installation with pressurised fluidised bed boiler
A – valve for air flow control, B – valve for pressure control

Lumped Parameter Model

The modelling problem at hand can be characterised as one concerning "complicated systems of machines and equipment." Of primary interest are transient processes resulting in changes of global thermal and flow parameters. To address such modeling problem an approach based on lumped parameter (zero-

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dimensional) models was developed in IHE. When applying this method, accumulation of mass and energy in flow part, being in reality continuously distributed in the flow space, is assigned to appropriately selected accumulating volumes. Therefore, in formulating the lumped parameter model, it is necessary to substitute the spatially finite object with separate elements possessing averaged properties – accumulating volumes. The mathematical description of the accumulating volume results from mass and energy balance equations:

$$\frac{dm}{dt} = \Sigma G_i \tag{1}$$

$$\frac{dH}{dt} = \Sigma G_i h_i + V\frac{dp}{dt} + Q \tag{2}$$

where: $m = \rho V$ – mass of the working fluid in the accumulating volume; G – mass flux; $H = \rho V h$ – total enthalpy of the working fluid in the accumulating volume; h – specific enthalpy; V – accumulating volume; Q – heat flux; p - pressure; t – time.

After converting equations (1) and (2) to the form dependent on chosen state variables of the modeled system (usually pressure and temperature of the working fluid in the volume) and separation of variables, the balance equations of the model (e.g. for the simplest accumulating volume Fig.2) assume the following form:

$$\frac{dT}{dt} = \frac{\frac{\partial\rho}{\partial p}G_1(h_1 - h_2) + (G_1 - G_2)\left(1 - \rho\frac{\partial h}{\partial p}\right)}{V\left[\rho\left(\frac{\partial h}{\partial T}\frac{\partial\rho}{\partial p} - \frac{\partial h}{\partial p}\frac{\partial\rho}{\partial T}\right) + \frac{\partial\rho}{\partial T}\right]} \tag{3}$$

$$\frac{dp}{dt} = \frac{G_1 - G_2}{V\frac{\partial\rho}{\partial p}} - \frac{\frac{\partial\rho}{\partial T}\left[\frac{\partial\rho}{\partial p}G_1(h_1 - h_2) + (G_1 - G_2)\left(1 - \rho\frac{\partial h}{\partial p}\right)\right]}{V\frac{\partial\rho}{\partial p}\left[\rho\left(\frac{\partial h}{\partial T}\frac{\partial\rho}{\partial p} - \frac{\partial h}{\partial p}\frac{\partial\rho}{\partial T}\right) + \frac{\partial\rho}{\partial T}\right]}$$

$$p_1 - p_2 = f(G_1, \rho_1);\ p = p_2;\ T = T_2;\ \rho = \rho_2,\ h = h_2 \tag{4}$$

where: $\frac{\partial\rho}{\partial p}, \frac{\partial\rho}{\partial T}, \frac{\partial h}{\partial T}$ are thermodynamic derivatives for the working fluid.

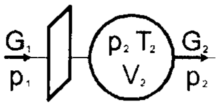


Fig.2 Scheme of the exemplary accumulating volume

Realization of Nonlinear Model

Selected accumulation volume coincide with containers of the flow system Fig.1. Ordinary differential equations describing mass and energy accumulation process in the volume can be written in matrix form $Ax=b$, exemplary for coal hopper (symbols like in Fig.3):

$$A = \begin{bmatrix} v\frac{\partial\rho}{\partial T} & v\frac{\partial\rho}{\partial P} & 0 & 0 & \rho \\ v_i\frac{\partial\rho}{\partial T} + v\rho\frac{\partial i}{\partial T} & v_i\frac{\partial\rho}{\partial P} + v\frac{\partial i}{\partial P} - V & m_w c_w & 0 & \rho_i - P \\ 0 & 0 & 0 & 0 & -\rho_w \\ 0 & 0 & m_w c_w & 0 & 0 \\ 0 & 0 & 0 & m_m c_m & 0 \end{bmatrix}$$

$$x = \begin{Bmatrix} \frac{dT_2}{dt} \\ \frac{dP_2}{dt} \\ \frac{dT_w}{dt} \\ \frac{dT_{1m}}{dt} \\ \frac{dv_2}{dt} \end{Bmatrix}, \quad b = \begin{Bmatrix} a_2 G_2 \\ a_2 G_2 t_2 - G_w c_w T_w + Q_2 - Q_3 \\ -G_w \\ -G_w c_w T_w + Q_3 \\ Q_1 - Q_2 \end{Bmatrix} \tag{5}$$

Differential equations are supplemented by a set of non-linear algebraic equations describing flow resistance, heat flow Q exchanged among environment, steel vessel and pulverised material, as well as properties of working medium. As a result one obtains the mathematical model of the system in a form of a set of differential-algebraic equations (DOEs):

$$\overline{F}_s\left(\frac{d\overline{X}}{dt},\ \overline{X},\ \overline{W},\ \overline{A}\right) = 0$$

$$\overline{F}_w\left(\overline{X},\ \overline{Y},\ \overline{W},\ \overline{A}\right) = 0 \tag{6}$$

where: $\overline{F}_s$ – system of ordinary differential equations (equations of state); $\overline{F}_w$ – system of algebraic equations (equations of output); $\overline{X}$ – vector of state coordinates; $\overline{W}$ – vector of input variables; $\overline{Y}$ – vector of output variables; $\overline{A}$ – vector of model coefficients.

Positions of the regulating valves A and B (Figs.1 and 3) (input variables u_1 and u_2 of the linear model) are the elements of the vector of input variables $\overline{W}$, air (exhaust gases) pressure behind the boiler (y_2) is one of the state variables $\overline{X}$, and the air mass flux behind the

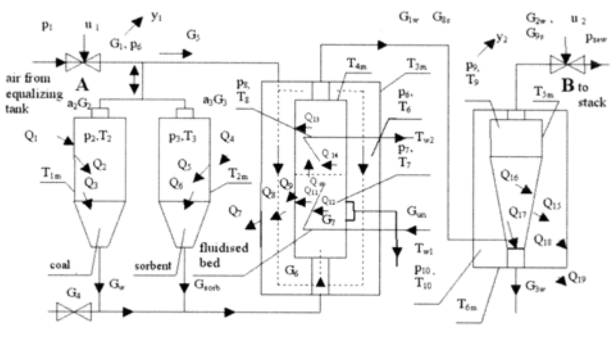


Fig.3 Scheme of the installation for modeling
A – valve for air flow G_1 - y_1 control,
B – valve for pressure p_9 - y_2 control

valve A (y_1) is an element of the output vector $\bar{Y}$.

The systems of equations (6) were solved using the fourth-order Runge-Kutta method. The derivates of inputs u_1 (t), u_2 (t) were approximated by finite differences.

Because of the complicated form of equations (5) and (6) there exists no possibility of finding a direct, analytical relationship between the outputs y_1, y_2 and the inputs u_1, u_2 of the linear model. Such a relationship can only be found numerically – for pre-set values of all variables, or experimentally. The experimental approach is discussed below.

Linear Model

As a result of the scientific collaboration of IHE and the Institute of Computer Science of the Warsaw University of Technology, the concept of applying the recent results in identification and computer control of multi-degree-of-freedom (multivariable) systems has emerged. In the Institute of Computer Science a novel method for the design of control systems in industrial plants - MIKOZ[1] – had been developed. The method can be applied to the wide range of continuous industrial processes, typical for power, chemical and petrochemical engineering. The method seems to be well suited to modeling processes in pressurized, fluidized bed. Control systems based on the application of the MIKOZ method significantly improve the performance of the plant. They ensure considerably higher quality of control in comparison to typical industrial automation systems, including those marketed by renowned companies. As a result a plant can operate closer to the set point, thus increasing efficiency and saving materials and energy.

The integral element of the MIKOZ method is experimental determining of object transfer matrix. It is achieved by applying at object inputs carefully designed test signals $u(t)$ and then appropriate analysis of outputs $y(t)$.

The basic principle of identification measurements is illustrated in Fig.4. The excitation $z(t)$ is applied from the front or back of the system and the time curves of the outputs $y(t)$, errors $e(t)$, and control signals $u(t)$ are recorded . The excitation $z(t)$ has a form of a carefully designed sine wave consisting of a number of harmonics with a certain base frequency (multi-tone testing) – Fig.5. The ratio of Fourier transforms of the output and the input provides points on the Bode diagram of the object, corresponding to the frequencies present in the excitation. Bode plot is then used to determine the transfer matrix of the object. The analytical form of the elements of the transfer matrix is given by equation (7)[3]:

$$G_{11} = \frac{2}{(1+2.5s)} e^{-35}$$

$$G_{12} = \frac{-3(1+s)^3}{(1+35s)(1+2s)^3} e^{-3.5s}$$

$$G_{21} = \frac{0.28}{(1+20s)(1+1.4s)} e^{-3.5s}$$

$$G_{22} = \frac{1}{(1+20s)(1+3s)} e^{-2.5s} \tag{7}$$

Fig.6 presents the comparison of the experimental and theoretical (equations (7)) amplitude and phase spectra. In the authors' opinion the agreement is very good.

The transfer matrix (7) constitutes the frequency

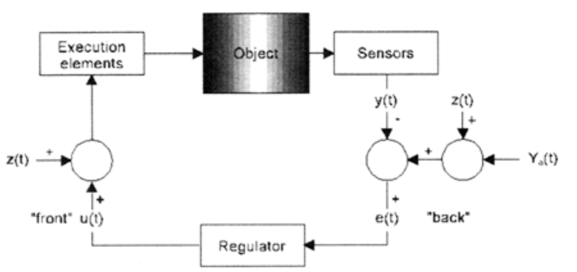
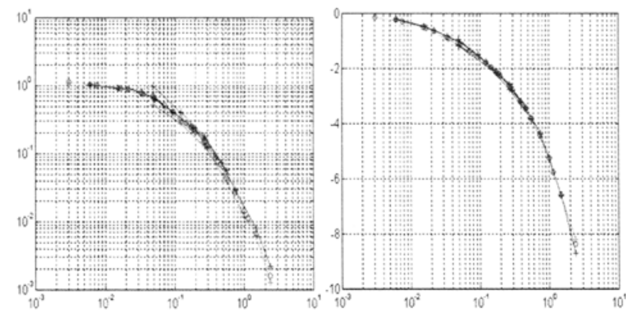
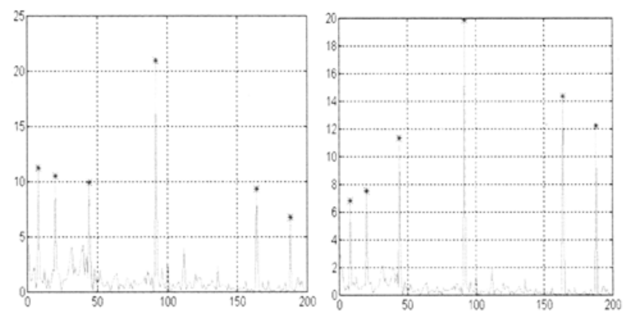


Fig.4 Principle of identification measurements
y(t) – vector of measured variables (outputs), $Y_o(t)$ – vector of set values, $e(t)$ – vector of errors, $u(t)$ – vector of control signals (inputs), $z(t)$ – excitation (only for one variable)



(a) module of G_{22} transfer function (b) phase of G_{22} transfer function

Fig.5 Exemplary result of identification



Number of harmonic Number of harmonic

(a) spectrum of u_1 (b) spectrum of y_1 (in linear scale of pulsation)

Fig.6 Exemplary result of identification measurement

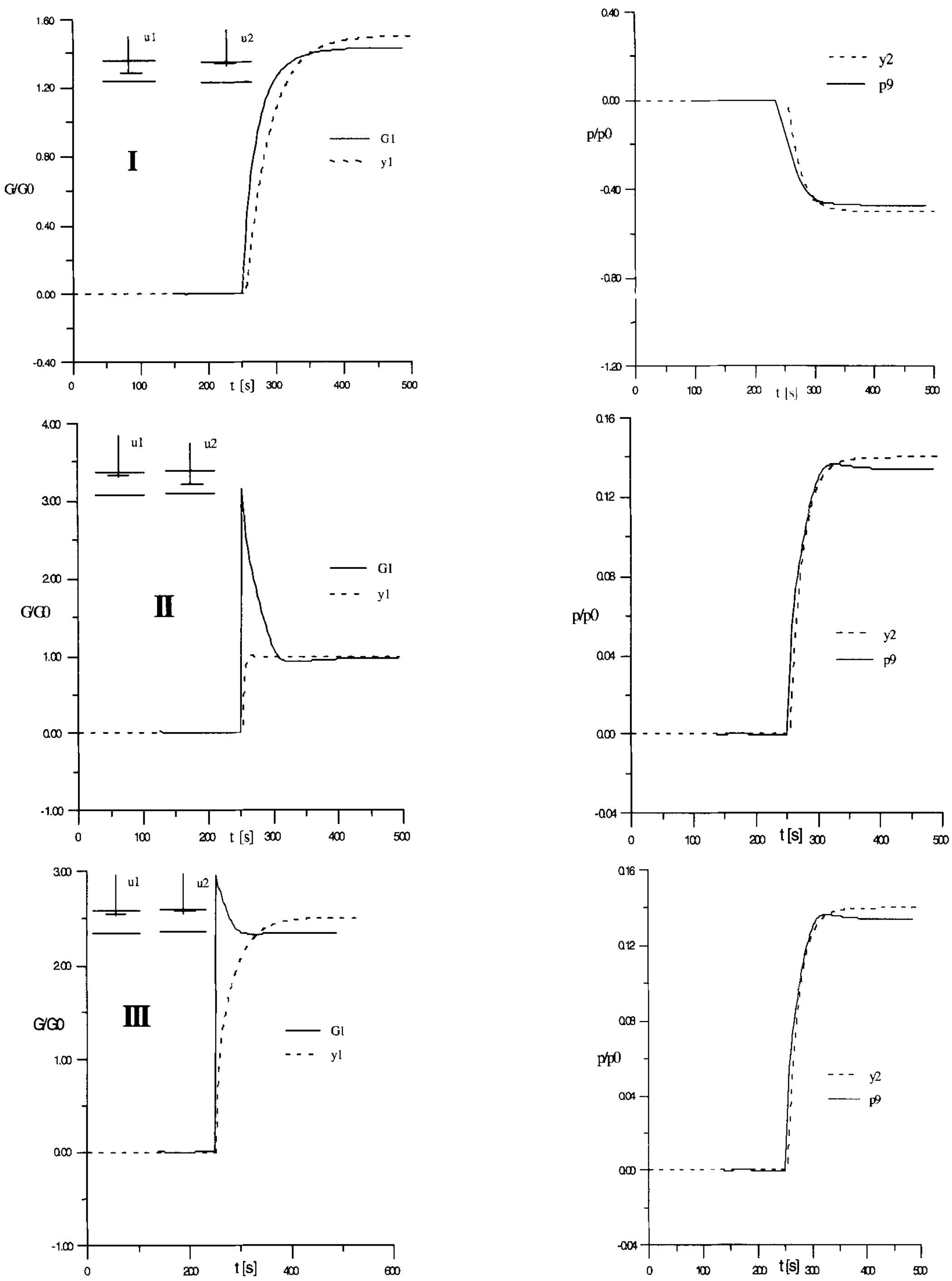


Fig.7 Time courses of relative air mass flow rate G1 at inlet G/G_0–y_1 and pressure p_9 behind the boiler p/p_0–y_2 as the result of simulations
G_1, p_9 from nonlinear model; y_1, y_2 – from linear model
Position of regulating valves u_1, u_2 are shown at top part of drawings
Initially both valves are at half open position
At time point of 250 s: I – the second valve u_2 jumps to full open position;
II – the first valve u1 jumps to full open position; III – both valves jump to full open position

domain mathematical model of the object, useful for designing control algorithms. On the other hand, equations (7) are equivalent to the system of linear, ordinary differential equations in time domain (8). They provide the relationship between the outputs (y_1, y_2) and the (time-shifted) inputs (u_1, u_2):

$$y_1(t)+43.5\frac{d}{dt}y_1(t)+324.5\frac{d^2}{dt^2}y_1(t)+$$
$$983\frac{d^3}{dt^3}y_1(t)+1350\frac{d^4}{dt^4}y_1(t)+700\frac{d^5}{dt^5}y_1(t)=$$
$$2u_1(t-3)+82\frac{d}{dt}u_1(t-3)+444\frac{d^2}{dt^2}u_1(t-3)+$$
$$856\frac{d^3}{dt^3}u_1(t-3)+560\frac{d^4}{dt^4}u_1(t-3)-$$
$$3u_2(t-3.5)-16.5\frac{d}{dt}u_2(t-3.5)-31.5\frac{d^2}{dt^2}u_2(t-3.5)-$$
$$25.5\frac{d^3}{dt^3}u_2(t-3.5)-7.5\frac{d^4}{dt^4}u_2(t-3.5)$$
$$y_2(t)+44.4\frac{d}{dt}y_2(t)+580.2\frac{d^2}{dt^2}y_2(t)+$$
$$1928\frac{d^3}{dt^3}y_2(t)+1680\frac{d^4}{dt^4}y_2(t)=$$
$$0.28u_1(t-3.5)+6.44\frac{d}{dt}u_1(t-3.5)+16.8\frac{d^2}{dt^2}u_1(t-3.5)+$$
$$u_2(t-2.5)+21.4\frac{d}{dt}u_2(t-2.5)+28\frac{d^2}{dt^2}u_2(t-2.5) \qquad (8)$$

Simulation Results

In order to evaluate and compare both modelling techniques, a number of computer simulations for the same inputs and initial states were conducted. Positions of regulating valves A (u_1) and B (u_2) were chosen as forcing inputs. Air flows G_1–y_1 and pressures in the system p_9–y_2 were chosen as model outputs. Obviously, the non-linear model is also able to provide results for all other system parameters. However, these additional results are not presented here. Examples of simulation results are given in Fig.7 and Fig.8.

Conclusions

The obtained time courses of examined parameters of the flow system are closed together-practically equivalent, despite of totally different mathematical forms of mathematical models related. It seems that in the domain of its applicability results obtained using both models can be treated as equivalent approximations of object properties.

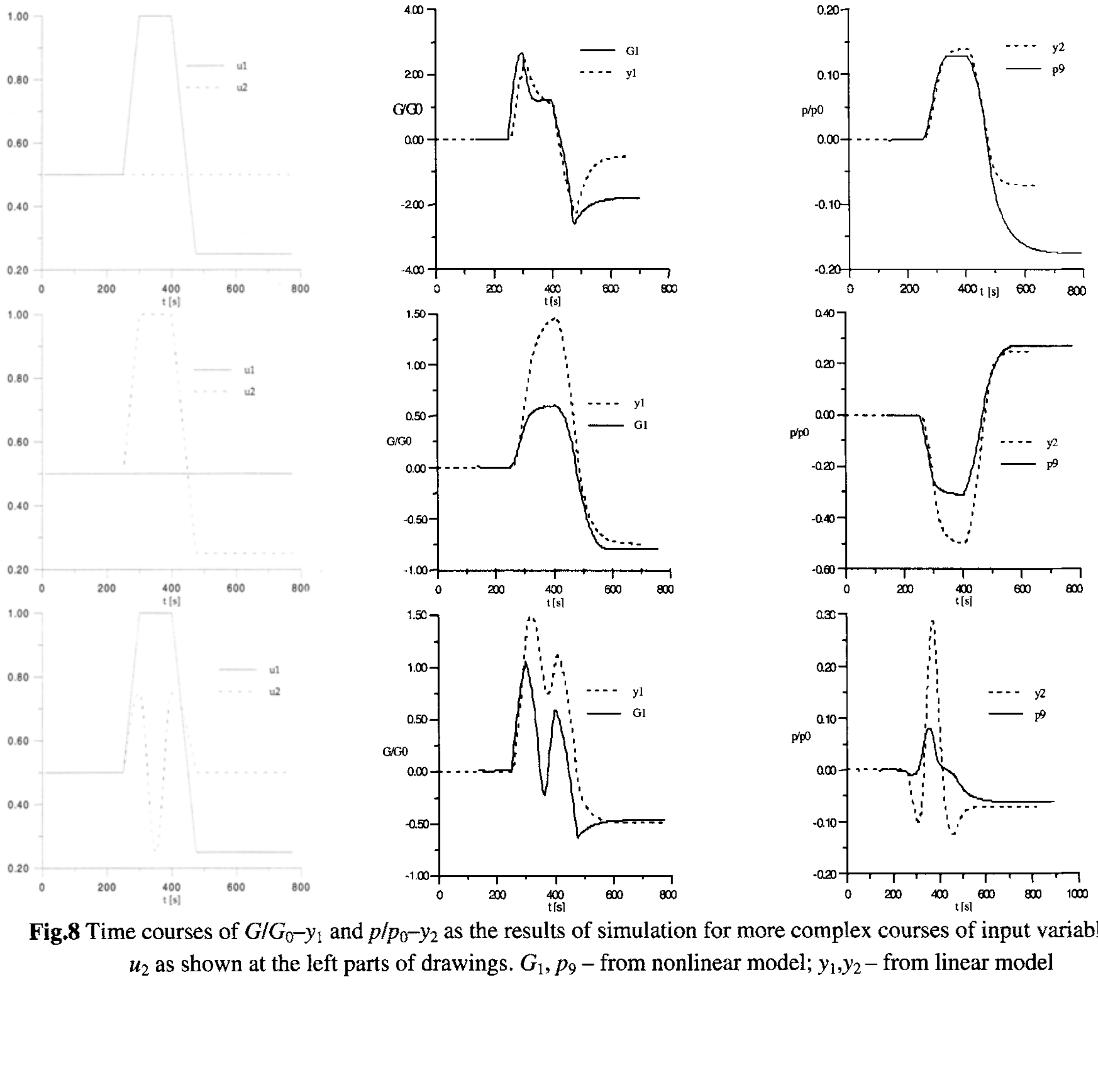


Fig.8 Time courses of G/G_0–y_1 and p/p_0–y_2 as the results of simulation for more complex courses of input variables u_1, u_2 as shown at the left parts of drawings. G_1, p_9 – from nonlinear model; y_1,y_2 – from linear model

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