

# Experimental Verification of Modeling of DELTA Robot Dynamics by Direct Application of Hamilton's Principle.

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## Abstract

*This contribution presents the experimental verification of the newly developed Hamilton-based dynamic model of DELTA direct drive parallel robot by measurements of DELTA robot's motor torques. The strain gage technology was applied. The experiment is in author's opinion the first ever for robot DELTA. The experimental results validate the modelling of robot DELTA as a system of rigid bodies, connected by frictionless joints. The results obtained from the model based on the direct application of Hamilton's Principle in extended space, taking into account the differences between real and desired accelerations, agreed with measurements to at least 5%.*

*The motor torque measuring system may be in future used in force control of DELTA robot.*

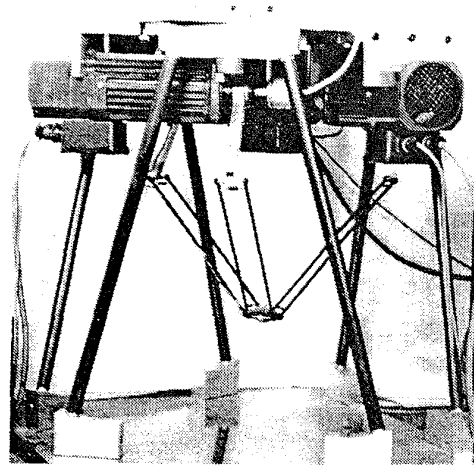
## 1 Introduction

### 1.1 Brief overview of parallel robots

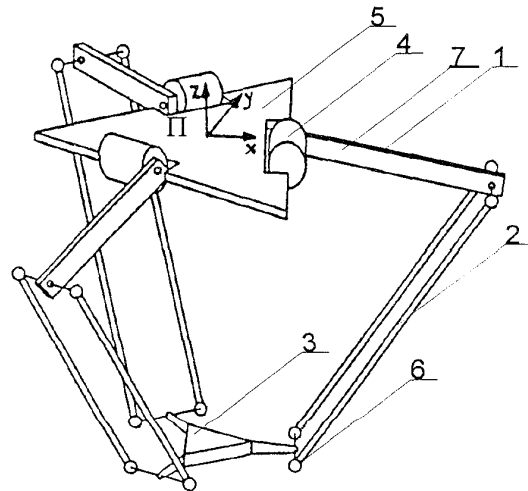
Research into the field of parallel robots documented in the literature dates back to the year 1938, when Pollard [1] patented his mechanism for car painting. In 1947 Mc Cough proposed a 6-degree-of-freedom platform, which was later used by Stewart in his flight simulator [2]. Parallel manipulators are particularly suited to a number of typical industrial applications and have presented a lot of interest to various researchers over the years. In recent years several new structures and mechanisms have been proposed, developed for a variety of both established and novel applications, such as packaging, assembly etc. [3, 4, 5, 6, 7, 8, 9].

Parallel manipulators possess a number of advantages when compared to traditional serial arms. They offer generally a much higher rigidity and smaller mobile mass than their serial counterparts.

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a)



b)

Fig. 1. a) Image of the DELTA-580 direct drive robot  
(the sum of the lengths of an arm and a  
forearm equals 580 mm);  
b) General lay-out of the DELTA robot.

These features allow, potentially, a much faster and more precise manipulations. The direct drive DELTA parallel

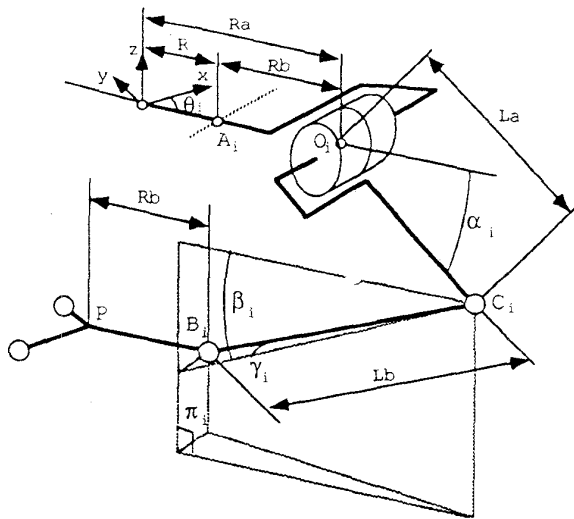


Fig 2. The dimensional parameters of the DELTA

robot designed by R. Clavel and co-workers at the Swiss Federal Institute of Technology, Lausanne, is capable of sustaining an acceleration better than  $500 \text{ m/s}^2$  ! Common to parallel mechanisms is their relatively reduced working volumes, which though smaller in comparison with serial arms of a similar size, are however usually more uniform and homogeneous, possessing no dead zones (unreachable regions). A catalogue possessing a large variety of parallel configurations can be found in the book by Merlet [10].

Fast and precise robot manipulation requires control algorithms that make the best use of the information extracted from the dynamics analysis of the robot (Feedforward [11], Computed Torque [11], Resolved Acceleration [12], Model-Reference Adaptive Control [13] and the references cited therein). Common to all these control approaches, except the last, is the problem linked to the solution of the inverse dynamics problem for the manipulators, and the challenge being to be capable of doing so in real time. This has necessitated an adapted numerical representation and a high computing efficiency. The DELTA parallel robot has been chosen as an application candidate for the model development as its intrinsic nature is representative of most of the complexities pertinent to the difficulties commonly encountered in the dynamic modelling of parallel structures [14, 15]. These include problematic issues such

as the complicated, spatial kinematic structure, which includes a number of unarticulated joints, the dominance of inertial and quadratic forces over the frictional and gravitational components, rapidly changing dynamics which places restrictions on the length of the sampling interval and hence limiting controller sophistication (i.e. the available inverse dynamics computation time), etc.. From the other hand, the inverse dynamic model to be used in the control scheme, must be of good accuracy.

Recently, the modelling approach based on the direct application of Hamilton's Principle has been proposed [15]. The aim of this study is to validate the modelling method and access its accuracy by measurement of DELTA robot's motor torques using strain gages. The described experiment is, to the best of author's knowledge, the first ever performed on the DELTA type robot.

## 1.2 DELTA Robot description

The DELTA robot (Fig 1) is an original design that arose from the need detected in the production and manufacturing sectors for manipulators better suited to the fast execution of light-duty tasks.

As seen from the figure the three closed kinematic chains that are identical and are actuated by three revolute electric motors (4 in Fig. 1b) rigidly mounted to the top (robot base, 5 in Fig. 1b), and closed below at the common tool-base (also known as the *travelling plate* or *nacelle*, 3 in Fig. 1b). The combination of the constrained motion of these three chains ensues in a resulting 3 translatory degrees of freedom for the robot tool-base. Each chain consists of an arm (1 in Fig. 1b) whose length represents a transmission factor amplification through the parallel rods (parallelogram, 4 in Fig. 1b) at the elbow. These rods transmit and convert the revolute actuator motion into the linear displacements of the tool-base. The 3 degrees of freedom obtained are the x, y, z displacements of the tool-base within the robot's working volume ; and its relative orientation is maintained fixed in an entirely passive manner. This structure is very rigid (approx. 120 Hz lowest natural frequency). Springs hold the ball-and-socket joints of the parallel rods at both the elbows and at the tool-base. Capable of achieving  $500 \text{ m/s}^2$ , the direct drive DELTA robot, to the best of our knowledge, is the fastest robot in the world.

The configuration currently under study is a symmetrical one and consists of the three chains between the robot base and the travelling plate, which are identical and arranged at 120 degrees to one another; the axes of the actuators of each arm are co-planar and tangential to the same circle. The arms describe planar

circles. The articulations (6 in Fig. 1b) that join the three pairs of parallel rods to the travelling plate are situated on the three sides of an equilateral triangle. Other configurations are possible but will not be touched upon here. From this point on, for simplicity's sake, the  $\Pi$  plane (Fig. 1b) will be considered horizontal and the planes in which the arms move to be vertical. The Tool Centre Point (TCP) is given by the intersection of three spheres whose origins are to be found at the arm's elbows. The dimensional parameters Ra, Rb, La, Lb and characteristic angles  $\alpha_i$ ,  $\Theta_i$ ,  $b_i$ ,  $\gamma_i$  are defined as in Fig. 2. The inverse as well as the direct kinematics problem for robot DELTA has a closed form analytical solution [3].

## 2 Models of DELTA robot dynamics

Dynamics of robots is usually modelled by multi-body system methods. The common assumptions made during modelling are the rigidity of links and the lack of friction in kinematic pairs (see for example [16, 17, 18]). For traditionally propelled, serial arms, the validity of this approach is doubtful. The presence of gears introduces high friction and considerable flexibility.

Since 1985, when Prof. Clavel introduced his design, a new family of robots with parallel kinematic structure has emerged. For robots belonging to this family, structurally more rigid than serial ones and powered by direct drive motors eliminating gears, the dynamics modelling approach based on the rigid body and no-friction assumptions seems to be of a good accuracy. The experimental verification of the above statement in the case of direct drive Delta robot has been the objective of the research described in this paper.

There are a few models of Delta robot dynamics, all treating the manipulator as a system of rigid bodies connected by frictionless kinematic pairs:

- Newton-Euler based [4],
- Lagrange based [14],
- Based on the direct application of the Hamilton's Principle [15].

The inverse problem of dynamics has been considered as the most interesting, since it has a straightforward application to model-based robot control [11]. The calculated motor torques for a given trajectory of the robot with the use of different dynamic models are obviously the same. Therefore results obtained using the Hamilton-based one [15], which is the most efficient numerically and does not require accelerations as input data has been utilised to compare with measurements.

The direct application of the Hamilton's Principle to Delta robot dynamics modelling results in derivation of the following set of finite difference equations [15]:

$$\left[ \left( \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right)_n + \sum_{j=1}^m \left( \lambda_j \left( \frac{\partial h_j}{\partial \dot{q}_i} \right)_n \right) + Q_i \right] \Delta t + \sum_{l=1}^N p_{il} \frac{\partial f_{il}}{\partial \dot{q}_i} = 0 \quad (1)$$

$$\left( \frac{\partial \mathcal{L}}{\partial q_i} \right)_n \Delta t + \sum_{l=1}^N p_{il} \frac{\partial f_{il}}{\partial q_i} = 0 \quad , \quad (2)$$

$$f_{in} = \Delta q_{in} - \dot{q}_{in} \Delta t = 0 \quad , \quad (3)$$

$$h_{jn}(q_i) = 0 \quad , \quad (4)$$

where: L=T-V Lagrange function,  
V - potential energy,  
T - kinetic energy,  
 $q_i$  - generalised co-ordinates,  
 $\{q_i\} = \{x, y, z, \alpha_1, \alpha_2, \alpha_3\}$   
 $p_{il}$  are conjugate momenta.  
 $f_{in}$  are the functions relating co-ordinates and velocities in each time instant,  
 $h_j$  - equations of constraints  
(geometrical model of the robot),  
 $Q_i$  - non-monogenic forces,  
 $\lambda_j$  - Lagrange multipliers,  
 $i=1, \dots, k$  - generalised co-ordinate counter,  
 $j=1, \dots, m$  - constraint counter,  
 $n=1, \dots, N$  - time interval counter.

Obtaining the kinetic and potential energy of the system as well as its derivatives is not difficult and may be found in references [14, 15]. Also the geometrical model of robot Delta is simple and may be found for example in [1, 14].

The above equations, in application to the inverse problem of dynamics, require fewer mathematical operations than previously proposed Newton-Euler and Lagrange-based models, moreover the Hamilton-based model does not require accelerations as input data for motor torque calculation [15].

Hamilton-based model's numerical efficiency allowed its implementation in the feedforward control scheme of the robot.

## 3 Motor torque measurement method

The experiments were performed at the Laboratory of Microengineering of the Swiss Federal Institute of

Technology in Lausanne, on the direct drive DELTA-580 robot (Fig. 1a), controlled by the network of three transputers T800 supervised by a PC-486 compatible computer (see Fig.3) [4].

In order to measure Delta robot's motor torques, the strain gage technology has been utilised [19]. Two double gages DK13C 6/350 were arranged in full Wheatstone's

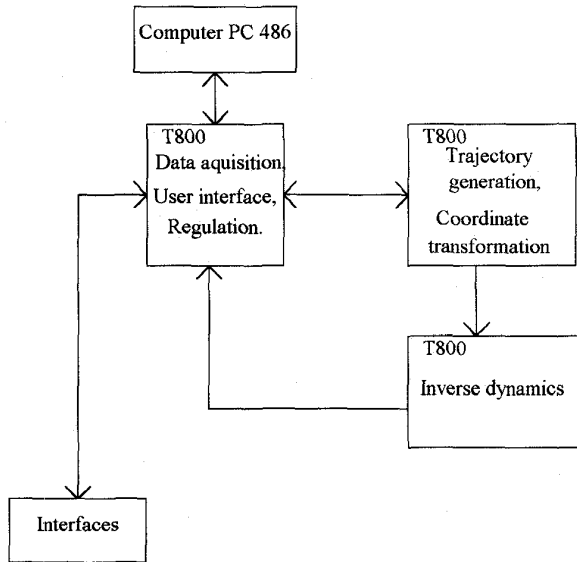


Fig. 3. Diagram of Delta robot control system.

bridge and attached to robots arm at 7 in Fig. 1. The electrical signal from the bridge was amplified by KWS 3073 amplifier delivered by HBM company. The analogue signal from the amplifier was converted to the digital form by HADC 574 Z BCD A/D converter from Signal Processing Technologies company. Afterwards the signal, passing specialised cards manufactured at Swiss Federal Institute of Technology in Lausanne, was acquired by the appropriate transputer and became available for analysis. The data acquisition was performed in real time, synchronously with position measurement and control signal dispatch.

The motor torques transfer onto the structure of the Delta robot through its arms. The Delta's arm has only one degree of freedom: the rotation with respect to the motor axis. Therefore, to find the motor torque it is sufficient to measure only one component of a bending torque in the robot's arm: the one acting in the arm's plane.

The accuracy of the measurement depends on the following factors:

- Accuracy of the measurement of the strain gages fixation cross-section from the motor axis, by a slide calliper with electronic display - 0.02 mm.
- Accuracy of the calibration - the response of the measuring system was much faster than changes of the measured torque. So that the calibration in static conditions has been satisfactory. The measuring system has been found to be linear. The calibration constant has been calculated by least square method. The masses of weights (1 - 4 kg) used during calibration has been known to within 0.1 g.
- Resolution of the 12 bit A/D converter -  $10[V] / 2047 = 0.004885 [V]$ . Taking into account the calibration constant, the resolution was 0.0088 Nm.
- Degree of the amplifier linearity.
- Accuracy of the assumed value of the terrestrial acceleration -  $9.81 \text{ m/s}^2$ .

It may be stated that the torque measurement has been performed with the accuracy of at least 0.05 Nm. For the purpose of the comparison with the theoretical predictions of the Hamilton based dynamic model, the achieved accuracy is more than satisfying.

#### 4 Torque measurement results

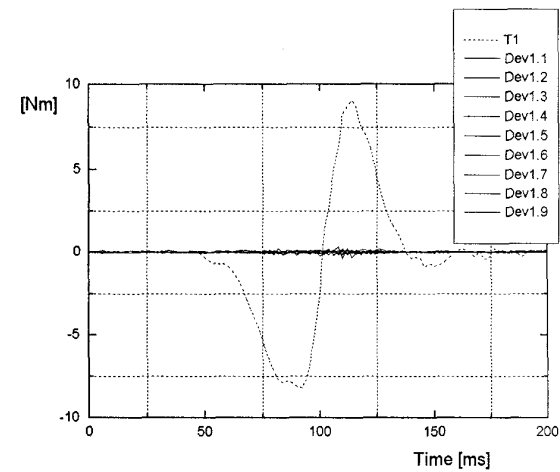
A number of trajectories was tested. Selected results for the fastest non-symmetrical elliptical trajectory are presented below. The acceleration of the travelling plate during the movements exceeded  $170 \text{ m/s}^2$ .

Fig. 4a shows the repeatability of measurements. The repeatability is satisfactory.

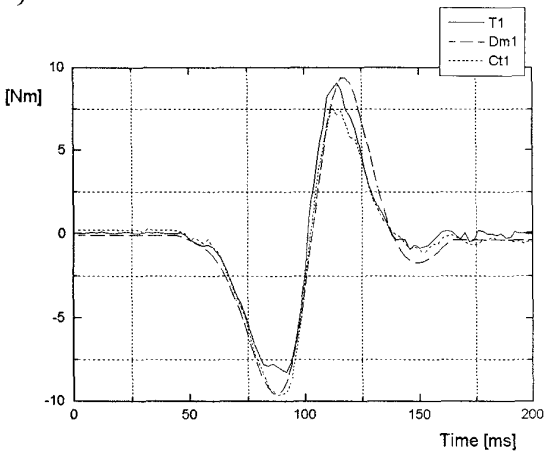
Fig. 4b presents the comparison of the measured torque (T1), the calculated one (Dm1), and the total regulator signal (Ct1). To calculate the torque, the model based on the direct application of Hamilton's Principle (Dm1) has been used. As input data the desired trajectory was adopted.

The agreement between the measured (T1) and calculated (Dm1) torque is good. The differences for the minimal and maximal torques results from the fact, that the robot, despite good agreement between desired and real position and velocity (used in feedback loop), failed to achieve maximal acceleration and deceleration (up to 10% difference for 5 ms period at max. deceleration).

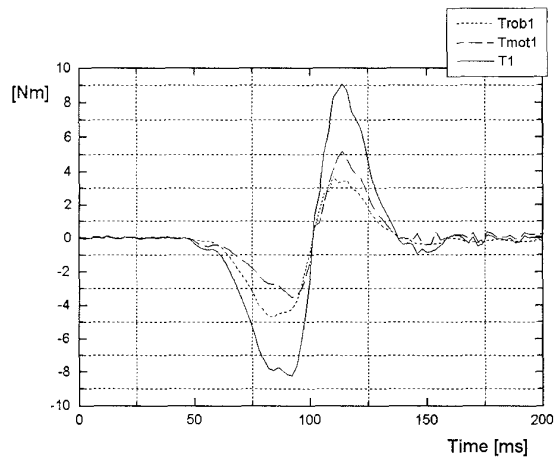
The total signal from the regulator (Ct1) has been presented to stress the difference between it and the measured torque (T1). These researchers, who without taking into account the non-linearities of electrical circuits and robot motors, treat the regulator signal as actually effected torque, may admit considerable mistakes in their judgement.



a)



b)



c)

T1 - measured motor torque for axis 1 [Nm], average from nine measurements.

Dev1.1,...,Dev1.9 - deviations from the average (T1) for nine measurements [Nm].

Dm1 - Torque calculated with the use of the model based on the direct application of Hamilton's Principle [Nm].

Ct1 - Total signal from the regulator [Nm].

Trob1 - Motor torque for axis 1 [Nm] moving only the structure of the robot (without the motor itself).

Tmot1 - Motor torque for axis 1 [Nm] moving only the robot's motor for axis 1 [Nm].

Fig. 4 Motor torque of robot Delta (axis 1):

a) repeatability;

b) comparison of calculated and measured torques;

c) comparison of the relative inertia of the manipulator and the motor.

Fig. 4c presents the comparison of the total motor torque (T1) and its components powering only the motor (Tmot1) and only the manipulator's structure (Trob1). It may be stated, that the inertia of robot Delta, unlike that of standard serial robots available on the market, is not dominated by the inertias of the motors.

The results for axes 2 and 3 are qualitatively similar to those for the axis 1.

## 5 Conclusions

In the paper the first direct measurement of motor torques for robot Delta, with the use of the strain gage technology is presented.

The results confirm the validity of the modelling approach based on the direct application of the Hamilton's Principle, and "rigid body" and "no-friction" simplifying assumptions for the direct drive DELTA robot. The results of the calculations, after taking into account the differences between the desired and real accelerations, agree with the measurements to at least 5%. Taking into consideration the necessity of inverse dynamics problem solving in real time, such a modelling accuracy is very good.

The measurement results show, that inertia of robot Delta, unlike that of standard serial robots available on the market, is not dominated by the inertias of the motors (see Fig. 4c).

The differences between the total signal from the regulator and the measured torque show, that these

researchers, who without taking into account the non-linearities of electrical circuits and robot motors, treat the regulator signal as actually effected torque, may admit considerable mistakes in their judgement (see Fig. 4c).

The motor torque measuring system may be in future used in force control of DELTA robot.

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